Flow Model For Open-Channel Reach Or Network

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CONTENTS

ILLUSTRATIONS

TABLE

INTERNATIONAL SYSTEM OF UNITS (SI) AND INCH-POUND SYSTEM EQUIVALENTS

SI unit	*Inch-pound equivalent*
Length	
centimeter (cm) =	0.3937 inch (in)
meter (m) =	3.281 feet (ft)
kilometer (km) =	0.6214 mile (mi)
Area	
centimeter^2 (cm^2) =	0.1550 inch^2 (in^2)
meter^2 (m^2) =	10.76 feet^2 (ft^2)
kilometer^2 (km^2) =	0.3861 mile^2 (mi^2)
Volume	
centimeter^3 (cm^3) =	0.06102 inch^3 (in^3)
meter^3 (m^3) =	35.31 feet^3 (ft^3)
=	8.107×10^{-4} acre-foot (acre-ft)
Volume per unit time	
meter^3 per second (m^3/s) =	35.31 feet^3 per second (ft^3/s)
=	1.585×10^4 gallons per minute (gal/min)
Mass per unit volume	
kilogram per meter^3 (kg/m^3) =	0.06243 pound per foot^3 (lb/ft^3)
gram per centimeter^3 (g/cm^3) =	6.243×10^{-5} pound per foot^3 (lb/ft^3)
Temperature	
degree Celsius (°C) =	(degree Fahrenheit − 32)/1.8 (°F)

SYMBOLS

Symbol	*Definition*
A	Area of conveyance part of cross section
B	Total top width of cross section
B_c	Top width of conveyance part of cross section
C_d	Water-surface drag coefficient
dA	A finite elemental area
f	A function
$f(I)$	Functional representation of a dependent variable
g	Gravitational acceleration
i	Subscript index that denotes a function's spatial location
j	Superscript index that denotes a function's temporal location
k	Flow-resistance coefficient function
q	Lateral flow per unit length of channel
Q	Flow discharge
Q_m	Flow discharge of mth branch at a junction
R	Hydraulic radius of cross section
$\boldsymbol{S}$	Vector of state
t	Time
Δt	Time increment
Δt_j	Time increment of jth interval
u	Flow velocity at a point
u'	X-component of lateral flow velocity
$\boldsymbol{u}$	Transformation matrix
$\boldsymbol{u}_{(i)}$	Transformation matrix of ith segment
$\boldsymbol{u}_n$	Transformation matrix of nth branch
U	Mean velocity of flow
$\boldsymbol{U}$	Transformation matrix
$\boldsymbol{U}_{(i)}$	Transformation matrix of ith segment
$\boldsymbol{U}_n$	Transformation matrix of nth branch
U_a	Wind velocity
W_k	Nodal flow at kth junction
x	Distance along channel thalweg
Δx	Distance increment
Δx_i	Distance increment of ith segment
Z	Water-surface elevation
Z_m	Water-surface elevation of mth branch at a junction
α	Angle between wind direction and x-axis
β	Momentum coefficient
γ	Flow-equation coefficient
δ	Flow-equation coefficient
ϵ	Flow-equation coefficient
ζ	Flow-equation coefficient
η	Flow-resistance coefficient similar to Manning's n
θ	Time weighting factor for spatial derivatives
λ	Flow-equation coefficient
μ	Flow-equation coefficient
ξ	Wind-resistance coefficient
ϱ	Water density
ϱ_a	Atmospheric density
σ	Flow-equation coefficient
χ	Weighting factor for function values
ψ	Space weighting factor for temporal derivatives
ω	Flow-equation coefficient
$\bar{}$	Superscript notation used to signify local constants

FLOW MODEL FOR OPEN-CHANNEL REACH OR NETWORK

By RAYMOND W. SCHAFFRANEK

ABSTRACT

Formulation of a one-dimensional model for simulating unsteady flow in a single open-channel reach or in a network of interconnected channels is presented. The model is both general and flexible in that it can be used to simulate a wide range of flow conditions for various channel configurations. It is based on a four-point (box), implicit, finite-difference approximation of the governing nonlinear flow equations with user-definable weighting coefficients to permit varying the solution scheme from box-centered to fully forward. Unique transformation equations are formulated that permit correlation of the unknowns at the extremities of the channels, thereby reducing coefficient matrix size and execution time requirements. Discharges and water-surface elevations computed at intermediate locations within a channel are determined following solution of the transformation equations. The matrix of transformation and boundary-condition equations is solved by Gauss elimination using maximum pivot strategy. Two diverse applications of the model are presented to illustrate its broad utility.

INTRODUCTION

In the past, the utility of numerical-simulation modeling was often limited by imposition of certain simplifying assumptions that were both necessary and justifiable at the time—necessary because numerical methods and (or) computer capacity were deficient and justifiable because parametric evaluation techniques and (or) equipment were lacking or inadequate. Today, for the most part, advances in numerical methods, computer technology, and hydrologic instrumentation have enabled model engineers to reduce the number of such restrictions, thus producing models that are more nearly formulated on pure hydraulic considerations and have a greater potential to provide more comprehensive flow information. Consequently, the scope and complexity of hydrodynamic problems that are now tractable have expanded.

This expansion of the role of numerical-simulation modeling has stimulated the need for rapid, economical, and efficient techniques to compile and appraise prototype data and model results. Thus, it is insufficient for a numerical scheme to be developed merely to the state of being a model program. To achieve a state of usefulness as an operationally oriented investigative tool, the model program must be supported by a comprehensive user-oriented data system and must provide a ready means of presenting output results in varied graphical forms.

In view of this need, the U.S. Geological Survey has developed a comprehensive, one-dimensional numerical-simulation model that is fully supported by a user-oriented system for modeling. The branch-network flow model, as it is called, is capable of simulating unsteady flow in a single open-channel reach or throughout a network of reaches composed of simply or multiply connected one-dimensional flow channels governed by various time-dependent forcing functions and boundary conditions. Operational modeling capability is achieved by linking the model to a highly efficient storage-and-retrieval module that accesses a data base containing time series of boundary values and by including an extensive set of digital graphics routines. These features help transform the model into a comprehensive tool for practical use in the conduct of hydrologic investigations.

Two illustrative applications of the model are presented. Application to a 274-m reach of Pheasant Branch near Middleton, Wis., demonstrates its capability in computing unsteady flow in short, upland-river reaches that can be highly responsive to climatological conditions. Application to a 25-branch schematization of the 50-km tidal river part of the Potomac Estuary near Washington, D.C., illustrates its feasibility in simulating tidal flows in estuarine-type network environments that are frequently subject to extreme freshwater inflows and variable meteorological influences.

Graphical capabilities of the model are also identified and discussed. One particular form of output is presented to illustrate how the model can be used to track and display the movement of neutrally buoyant conservative substances through a riverine system and thereby evaluate its transport and flushing properties.

TERMINOLOGY

To facilitate further discussion of the application of the model to either a single riverine channel or a system of channels, a few definitions are necessary. The terms "reach" and "branch" are used somewhat interchangeably to mean a length of open channel. The primary subdivision of a reach or branch is referred to as a "subreach" or "segment." A "network" is defined as a system of open channels either simply connected in treelike fashion or multiply connected in a configuration that permits more than one flow path to exist between certain locations in the system.

ONE-DIMENSIONAL UNSTEADY-FLOW EQUATIONS

One-dimensional unsteady flow in open channels can be described by two partial-differential equations expressing mass and momentum conservation. These well-known equations, frequently referred to as the unsteady flow, shallow water, or St. Venant equations (Baltzer and Lai, 1968; Dronkers, 1969; Strelkoff, 1969; Yen, 1973) can be written

$$B\frac{\partial Z}{\partial t}+\frac{\partial Q}{\partial x}-q=0 \quad (1)$$

and

$$\frac{\partial Q}{\partial t}+\frac{\partial(\beta Q^2/A)}{\partial x}+gA\frac{\partial Z}{\partial x}+\frac{gk}{AR^{4/3}}Q|Q| - qu'-\xi B_c U_a^2\cos\alpha=0, \quad (2)$$

in which the momentum coefficient, β, the flow-resistance function, k, and the wind-resistance coefficient, ξ, are defined as

$$\beta=\frac{1}{U^2A}\int u^2 dA, \quad (3)$$

$$k=\left(\frac{\eta}{1.49}\right)^2 \text{ (or, in SI units, as } k=\eta^2\text{)}, \quad (4)$$

and

$$\xi=C_d\frac{\varrho_a}{\varrho}\cdot \quad (5)$$

In these equations, formulated using water-surface elevation, Z, and flow discharge, Q, as the dependent variables, distance along the channel thalweg, x, and elapsed time, t, are the independent variables. (Longitudinal distance, x, and flow discharge, Q, are positive in the downstream direction.) Other quantities in the preceding equations are defined as follows:

A, area of conveyance part of cross section;
B, total top width of cross section;
B_c, top width of conveyance part of cross section;
C_d, water-surface drag coefficient;
g, gravitational acceleration;
q, lateral inflow per unit length of channel (negative for outflow);
R, hydraulic radius of cross section;
u, flow velocity at a point;
u', x-component of lateral flow velocity;
U, mean velocity of flow, $=Q/A$;
U_a, wind velocity;
α, wind direction measured from positive x-axis;
η, flow-resistance coefficient similar to Manning's n;
ϱ, water density; and
ϱ_a, atmospheric density.

Although hydraulic radius (R) is used in equation 2 and in subsequent expansions throughout this development, the commonly used substitution of hydraulic depth is employed in the model. This approximation ($R\approx A/B$) is assumed valid for shallow water bodies, that is, channels having a large width-to-depth ratio.

The momentum coefficient, β, also called the Boussinesq coefficient, is present in the equation of motion to account for any nonuniform velocity distribution. (See eq. 3.)

Equations 1 and 2 are, in general, descriptive of unsteady flow in a channel of arbitrary geometric configuration having both conveyance and overflow (or only conveyance) areas and potentially subject to continuous lateral flow and (or) the shear-stress effects of wind. In their formulation, it is assumed that the water is homogeneous in density, that hydrostatic pressure prevails everywhere in the channel, that the channel bottom slope is mild and uniform, that the channel bed is fixed (i.e., no scouring or deposition occurs), that the reach geometry is sufficiently uniform to permit characterization in one dimension, and that frictional resistance is the same as for steady flow, thus permitting approximation by the Chézy or Manning equation.

MODEL FORMULATION

Numerous varied mathematical methods and corresponding numerical schemes exist that render approximate solutions of the flow equations. However, new methods and alternative schemes that provide more accurate approximations and are inherently more flexible and efficient are continually being sought. In the branch-network model formulation, the flow equations are expressed in finite-difference form using a weighted four-point (box) scheme. This technique, also used by Fread (1974) and by Cunge and others (1980), permits the model to be applied using unequal segment lengths and box-centered to fully forward discretizations. A unique transformation operation is applied to the segment flow equations in the branch-network model, however, to lower the order of the coefficient matrices and thereby reduce computer time and storage requirements. A general matrix solution algorithm is used to simultaneously solve the resultant branch-transformation and boundary-condition equations. The implicit solution method is employed because of its inherent efficiency and superior stability properties. An optional iteration procedure, controllable by user-defined tolerance specifications, is additionally provided to permit improving the accuracy of the computed unknowns.

FINITE-DIFFERENCE TECHNIQUE

The space-time grid system shown in figure 1 depicts the region in which solution of the flow equations is sought. The symbols θ and ψ represent weighting factors used to specify the time and location, respectively, within the Δt_j time increment and Δx_i distance increment at which derivative and functional quantities are to be evaluated. The temporal and spatial derivatives of the functional value, $f(I)$, that denotes the dependent variables—stage (water-surface elevation) and discharge—are discretized, respectively, as follows:

$$\frac{\partial f(I)}{\partial t} \approx \frac{f_{i+1}^{j+1} + f_i^{j+1} - f_{i+1}^{j} - f_i^{j}}{2\Delta t},$$

$$\psi = 0.5,\ \Delta t_1 = \ldots = \Delta t_j = \ldots = \Delta t \qquad (6)$$

and

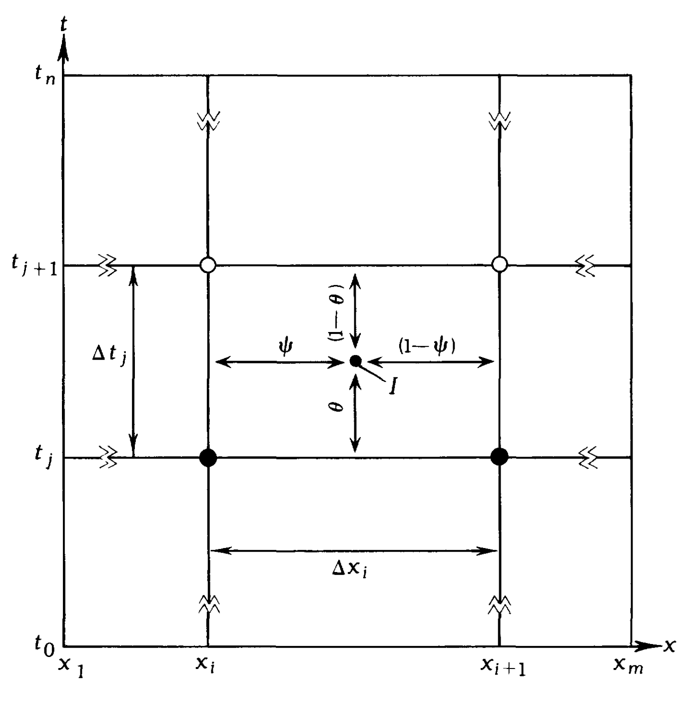

FIGURE 1.—Space-time grid system for finite-difference approximation.

$$\frac{\partial f(I)}{\partial x} \approx \theta \frac{f_{i+1}^{j+1} - f_i^{j+1}}{\Delta x_i} + (1-\theta) \frac{f_{i+1}^{j} - f_i^{j}}{\Delta x_i}. \qquad (7)$$

Usually θ is assigned in the range $0.5 \le \theta \le 1$. A value of 0.5 yields the fully centered scheme used by Preissmann (1960) and by Amein and Fang (1970), whereas a value of 1.0 yields the fully forward scheme presented by Baltzer and Lai (1968).

In a manner similar to treatment of the spatial derivatives, the cross-sectional area, top width, hydraulic radius, and discharges in nonderivative form in the equation of motion, denoted $f(I)$, are discretized as follows:

$$f(I) \approx \chi \frac{f_{i+1}^{j+1} + f_i^{j+1}}{2} + (1-\chi) \frac{f_{i+1}^{j} + f_i^{j}}{2}. \qquad (8)$$

Thus, these functional values can be represented on the same time level as the spatial derivatives or at any other different level within the time increment. The weighting factor χ may be assigned in the range $0 \le \chi \le 1$.

COEFFICIENT MATRIX FORMULATION

The partial-differential flow equations 1 and 2 are transformed into finite-difference expressions by application of the operators defined in equations 6–8 (Schaffranek and others, 1981). Using tilde ($\tilde{}$) notation to signify quantities taken as local constants, updated through iteration in the computation process, the equation of continuity can be reduced to

$$\gamma Z_{i+1}^{j+1} + Q_{i+1}^{j+1} + \gamma Z_i^{j+1} - Q_i^{j+1} = \delta, \qquad (9)$$

in which

$$\gamma = \frac{\tilde{B} \Delta x_i}{2\Delta t \theta}$$

and

$$\delta = \gamma \left(Z_{i+1}^{j} + Z_i^{j}\right) - \frac{(1-\theta)}{\theta} \left(Q_{i+1}^{j} - Q_i^{j}\right) + \frac{q \Delta x_i}{\theta},$$

and the equation of motion can be reduced to

$$Z_{i+1}^{j+1} + \zeta Q_{i+1}^{j+1} - Z_i^{j+1} + \omega Q_i^{j+1} = \epsilon, \qquad (10)$$

in which

$$\zeta = \lambda + \sigma + \mu,$$

$$\omega = \lambda + \sigma - \mu,$$

$$\lambda = \frac{\Delta x_i}{2\Delta t g \tilde{A} \theta},$$

$$\sigma = \frac{\chi k |\tilde{Q}| \Delta x_i}{2 \tilde{A}^2 \tilde{R}^{4/3} \theta},$$

$$\mu = \frac{2\beta \tilde{Q}}{g \tilde{A}^2},$$

and

$$\epsilon = -\frac{(1-\theta)}{\theta}\left(Z_{i+1}^j - Z_i^j\right) + \left(\lambda - \sigma\frac{(1-\chi)}{\chi}\right)\left(Q_{i+1}^j + Q_i^j\right) - \mu\frac{(1-\theta)}{\theta}\left(Q_{i+1}^j - Q_i^j\right) + \frac{\beta\tilde{Q}^2}{g\tilde{A}^3\theta}\left[\tilde{B}(\tilde{Z}_{i+1} - \tilde{Z}_i) + \left(\tilde{A}_{i+1} - \tilde{A}_i\right)\Big|_{\tilde{Z}_i}\right] + \frac{qu'\Delta x_i}{g\tilde{A}\theta} + \frac{\xi\tilde{B}_c\Delta x_i}{g\tilde{A}\theta}U_a^2 \cos\alpha.$$

Equations 9 and 10, which define the flow in the Δx_i segment, can then be expressed in the following matrix form:

$$\begin{bmatrix}\gamma & 1\\ 1 & \zeta\end{bmatrix}\begin{bmatrix}Z_{i+1}^{j+1}\\ Q_{i+1}^{j+1}\end{bmatrix} + \begin{bmatrix}\gamma & -1\\ -1 & \omega\end{bmatrix}\begin{bmatrix}Z_i^{j+1}\\ Q_i^{j+1}\end{bmatrix} = \begin{bmatrix}\delta\\ \epsilon\end{bmatrix}. \quad (11)$$

EQUATION TRANSFORMATION PROCEDURE

Equation 11 can be applied to all Δx_i segments within the network and the resultant equation set solved directly using appropriate boundary conditions and initial values. In the branch-network model, however, transformation equations are developed from the segment flow equations to correlate the unknowns at the ends of the branches, that is, at the junctions.

From a two-component vector of state for the ith cross section,

$$S_i^{j+1} = \begin{bmatrix}Z_i^{j+1}\\ Q_i^{j+1}\end{bmatrix},$$

the following transformation equation for the ith segment can be written

$$S_{i+1}^{j+1} = U_{(i)}S_i^{j+1} + u_{(i)}, \quad (12)$$

in which S_{i+1}^{j+1} is the vector of state for the $(i+1)$th cross section. The transformation matrices of the ith segment, $U_{(i)}$ and $u_{(i)}$, in which the subscript (i) denotes the segment, follow from the previously defined coefficient matrices:

$$U_{(i)} = \begin{bmatrix}\gamma_{(i)} & 1\\ 1 & \zeta_{(i)}\end{bmatrix}^{-1}\begin{bmatrix}-\gamma_{(i)} & 1\\ 1 & -\omega_{(i)}\end{bmatrix}$$

and

$$u_{(i)} = \begin{bmatrix}\gamma_{(i)} & 1\\ 1 & \zeta_{(i)}\end{bmatrix}^{-1}\begin{bmatrix}\delta_{(i)}\\ \epsilon_{(i)}\end{bmatrix}.$$

Successive application of the segment-transformation equation 12 to all segments contained in a branch results in an expression that relates the unknowns at the end cross sections 1 and m of the nth branch,

$$S_m^{j+1} = U_n S_1^{j+1} + u_n. \quad (13)$$

The transformation matrices of the nth branch, U_n and u_n, in which the subscript n denotes the branch, are obtained through successive substitution of the segment-transformation equation from the $(m-1)$th segment down to the first segment. These branch-transformation matrices,

$$U_n = U_{(m-1)}U_{(m-2)} \ldots U_{(1)} \quad (14)$$

and

$$u_n = u_{(m-1)} + U_{(m-1)}\,(u_{(m-2)} + U_{(m-2)}\,(u_{(m-3)} \ldots + U_{(3)}\,(u_{(2)} + U_{(2)}u_{(1)}) \ldots), \quad (15)$$

describe the relationship between the vectors of state, S_1^{j+1} and S_m^{j+1}, at the end cross sections of the branch, that is, at the junctions.

After applicable boundary-condition equations are formulated, the resultant equations are solved simultaneously, yielding stages and discharges at the termini of the branches (at the junction cross sections). Intermediate values of the unknowns at the internal segment ends (at cross sections between junctions) are subsequently determined through successive solution of the segment-transformation equation 12. This transformation procedure effects significant reductions in the model's requirements for computer memory and execution time. For example, if segment flow equations are used, a network consisting of N sequentially connected branches, each composed of M_i segments, would form a coefficient matrix of minimum order $2M+2$, where M is the total number of segments in the network, that is, the sum of the M_i's for the N-branch system. By combining segments into branches and using branch-transformation equations instead of segment flow equations, the size of the coefficient matrix can be reduced to order $4N$.

BOUNDARY CONDITIONS

To solve the branch-transformation equations implicitly, boundary conditions must be specified at *internal junctions* located at branch confluences within the network as well as at *external junctions* located at the extremities of branches, for example, where branches physically terminate or are delimited for modeling purposes. Equations describing the boundary conditions at internal junctions are automatically generated by the model, whereas boundary-condition equations for external junctions are formulated by the model from user-supplied time-series data or from user-specified functions.

Discharge and stage compatibility conditions can be expressed for internal junctions by neglecting velocity-head differences and turbulent energy losses. At a junction of n branches, discharge continuity requires that

$$\sum_{m=1}^{n} Q_m = W_k, \quad (16)$$

where W_k is zero or some user-specified external flow (inflow or outflow) at junction k, and stage compatibility requires that

$$Z_m = Z_{m+1},\ m = 1,\ 2,\ \ldots,\ (n-1). \tag{17}$$

Various combinations of boundary conditions can be specified for external junctions. A null discharge condition (as, for example, at a dead-end channel), known stage or discharge as a function of time, or a known, unique stage-discharge relationship can be prescribed.

Together, the internal and external boundary conditions provide a sufficient number of additional equations to satisfy requirements of the solution technique.

SOLUTION METHOD

The solution process begins at time t_0 by use of specified initial conditions and proceeds in Δt time increments to the end of the simulation at time t_n. Gauss elimination using maximum pivot strategy is employed to solve the system of equations. Iteration within a time step is performed to provide results within user-specified tolerances. The primary effect of iteration is to improve on the quantities taken as local constants within the time step, which in turn increases the accuracy of the computed unknowns. User-defined accuracy requirements are typically achieved in two or fewer iterations per time step.

MODEL APPLICATIONS

The thoroughness of the equation formulation on which a model is based largely governs the range of complexity of flows it can accommodate. The choice of numerical computation scheme primarily determines whether or not the model will be stable, convergent, accurate, and computationally efficient given that it is correctly and precisely implemented. However, for any model to be useful it must be subsequently transformed into a functional user-oriented simulation system, and its accuracy, reliability, and versatility must be adequately proved and demonstrated.

The branch-network model is being used to simulate the time-varying flows of several coastal and upland water bodies, as identified in table 1. These represent a broad spectrum of hydrologic field conditions, depicting such diverse hydraulic and field situations as hydropower-plant-regulated flows in a single upland-river reach, tide-induced flows in riverine and estuarine reaches and networks, unsteady flow in a residential canal system, and meteorologically generated seiches and wind tides in a multiply connected network of channels joining two large lakes.

Four types of model application are identified in table 1. The simplest of these is the single-branch type, which

TABLE 1.—*Applications of the branch-network flow model*

State	Water body location	Application type
Alabama	Coosa River near Childersburg.	35.2-km multiple branch.
	Alabama River near Montgomery.	21-branch multiconnected network.
Alaska	Knik/Matanuska River Delta near Palmer.	20-branch multiconnected network.
California	Sacramento River from Sacramento to Freeport.	17.4-km single branch.
	Sacramento River from Sacramento to Hood.	34.3-km multiple branch.
	Sacramento Delta between Sacramento and Rio Vista.	24-branch multiconnected network.
	Threemile Slough near Rio Vista.	5.2-km single branch.
Connecticut	Connecticut River near Middletown.	9.8-km single branch.
	Connecticut River downstream from Hartford.	41.2-km multiple branch.
Florida	Cape Coral residential canal system.	16-branch multiconnected network.
	Peace River from Arcadia to Fort Ogden.	30-km multiple branch.
	Peace River from Fort Ogden to Harbour Heights.	21-branch multiconnected network.
Idaho	Kootenai River near Porthill.	54.8-km multiple branch.
Kentucky	Ohio River downstream from Greenup Dam.	21.7-km single branch.
Louisiana	Atchafalaya River near Morgan City.	8-branch multiconnected network.
	Wax Lake Outlet near Calumet.	15-branch multiconnected network.
	Calcasieu River between Lake Charles and Moss Lake.	13-branch multiconnected network.
	Quachita River from Monroe to Columbia.	78.9-km multiple branch.
	Vermillion River from Lafayette to Perry.	48.3-km multiple branch.
	Loggy Bayou near Ninock.	9.2-km single branch.
	Mermentan River from Mermentan to Lake Arthur.	25.7-km multiple branch.
Maryland	Potomac River near Washington, D.C.	25-branch multiconnected network.

TABLE 1.—*Applications of the branch-network flow model*—Continued

State	Water body location	Application type
Michigan	Detroit River near Detroit.	12-branch multiconnected network.
	Saginaw River near Saginaw.	14-branch dendritic network.
Missouri	Osage River near Schell City.	2.6-km single branch.
New York	Hudson River from Albany to Poughkeepsie.	9-branch dendritic network.
N. Dakota	Red River of the North at Grand Forks.	1.3-km single branch.
S. Carolina	Intracoastal Waterway near Myrtle Beach.	36.7-km multiple branch.
	Cooper River at Diversion Canal.	6.3-km single branch.
	Cooper River at Lake Moultrie Tailrace.	1.5-km single branch.
	Back River near Cooper River confluence.	2.2-km single branch.
S. Dakota	James River near Hecla.	8.5-km multiple branch.
Washington	Columbia River downstream from Rocky Reach Dam.	3.1-km single branch.
Wisconsin	Pheasant Branch near Middleton.	0.27-km single branch.
	Menomonee River near Milwaukee.	0.61-km single branch.
	Milwaukee Harbor at Milwaukee.	12-branch multiconnected network.

is an application to a single reach of channel delimited by a pair of external boundary conditions. The multiple-branch type is an application to a channel, again delimited by a pair of external boundary conditions, but schematized as a series of sequentially connected reaches. The dendritic-network type is an application to a channel system composed of branches connected in treelike fashion. The multiply connected network type is likewise an application to a channel system, but one in which the branches are interconnected, thereby permitting multiple flow paths between certain locations in the system.

To illustrate the diverse capabilities of the model, two applications identified in table 1 are discussed briefly herein. These particular applications were selected to demonstrate the flexibility of the model in accommodating a wide range of hydrologic conditions and field situations.

PHEASANT BRANCH NEAR MIDDLETON, WIS.

Pheasant Branch is a tributary to Lake Mendota near the city of Middleton in Dane County, Wis. A 5-year study has been conducted by the U.S. Geological Survey, in cooperation with the city of Middleton and the Wisconsin Geological and Natural History Survey, to determine the sediment transport, streamflow characteristics, and stream-channel morphology in the Pheasant Branch drainage basin. In support of this effort, a short reach of Pheasant Branch was modeled to provide data on streamflow. Backwater effects from the lake and storm-generated transient flows necessitate use of an unsteady flow model.

The Pheasant Branch reach, which begins in marshland and ends 274 m downstream at Lake Mendota, is treated as a single segment in the model. Under typical flow conditions, the channel is on the order of 6.5 m wide, with a maximum depth of 1.5 m.

Water-surface elevations used as boundary conditions (identified as station 05–4279.52 at the upstream end and 05–4279.53 at the downstream end) for simulating flow in Pheasant Branch during June 15–21, 1978, are shown in the upper part of figure 2. Discharges computed by the model at the upstream end of the reach are illustrated in the lower part of figure 2. Some rapid oscillations in the boundary-value data, identified as noise caused by wind-generated waves, are discernible, particularly in the stage hydrograph recorded at the downstream end of the reach near Lake Mendota. These oscillations in the boundary-value data are reflected and accentuated in the hydrograph of computed discharges.

The Pheasant Branch model results plotted in figure 2 were computed using a 15-minute time step. The weighting factors θ and χ were set at 0.8. A value of 0.0385 was used for η, and the momentum coefficient, β, was assigned a value of 1.0. These parameter values were determined in model calibration tests conducted using discharges measured at the mouth of Pheasant Branch during other flow periods.

The June 15–21 simulation required 1.6 CPU seconds to complete on an Amdahl 470/V7[1] computer. The model required less than one (0.9) iteration per time step, on the average, during the simulation.

The computed results indicate that this stream is extremely responsive and sensitive to changing climatological conditions; therefore, these factors must be accurately represented by the model.

POTOMAC RIVER NEAR WASHINGTON, D.C.

In October 1977, the Water Resources Division of the U.S. Geological Survey instituted a 5-year interdisciplinary study of the tidal Potomac River and Estuary (Callender and others, 1984). The research areas undertaken in this investigation included historical geologic

[1] Use of firm or trade names in this report is for identification purposes only and does not constitute endorsement by the U.S. Geological Survey.

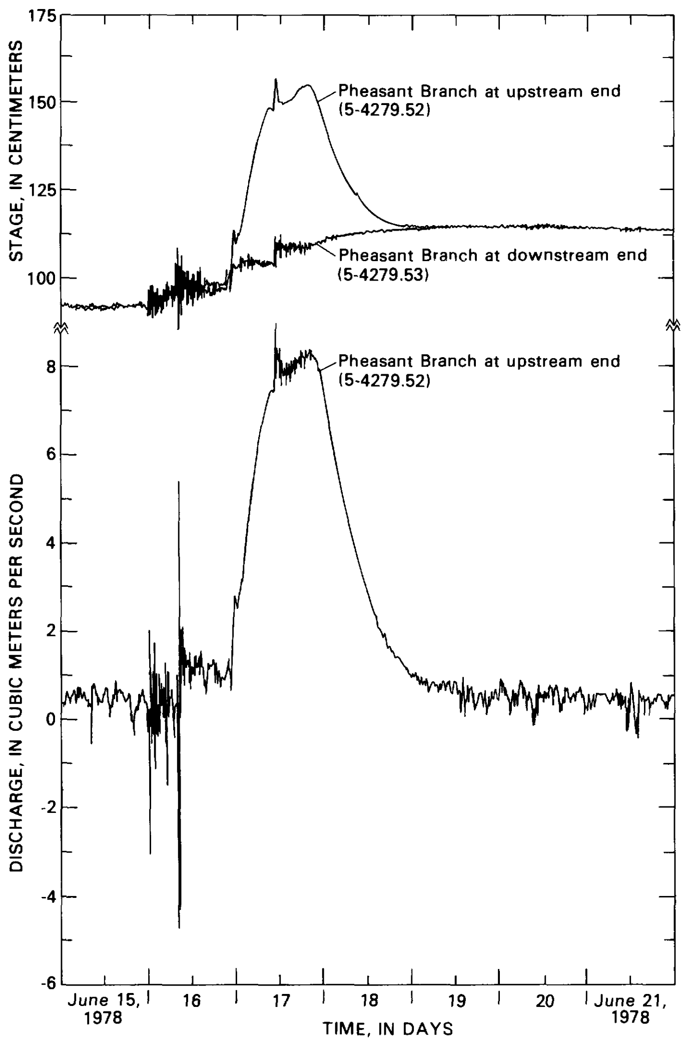

FIGURE 2.—Measured water-surface elevations and computed discharges for Pheasant Branch near Middleton, Wis.

studies, geochemistry of bottom sediments, nutrient cycling, sediment transport and tributary loading, wetland studies, benthic ecology, and hydrodynamics. The objective of the hydrodynamics project was to devise, implement, calibrate, and verify a series of numerical flow/transport simulation models in support of the other research efforts. To quantify the hydrodynamics of the tidal river, the branch-network model was applied to the 50-km segment of the Potomac, including its major tributaries and inlets from the head of tide at the fall line in the northwest quadrant of Washington, D.C., to Indian Head, Md., as shown in figure 3.

The Potomac River downstream from Chain Bridge is confined for a short distance (approximately 5 km) to a narrow, deep, but gradually expanding channel bounded by steep rocky banks and high bluffs. Farther downstream the river consists of a broad, shallow, and rapidly expanding channel confined between banks of low to moderate relief. Seven cross-sectional profiles illustrating the channel geometry are plotted in figure 3. The cross-sectional area and corresponding channel width expand more than fortyfold between Chain Bridge and Indian Head. In general, the depth varies from about 9 m at Chain Bridge to about 12 m at Indian Head.

Flow in the upstream portion of the tidal river is typically unidirectional and pulsating; bidirectional flow occurs in the broader downstream portion. The location of the transition from one flow pattern to the other varies, primarily in response to changing inflow at the head of tide but also to changing tidal and meteorological conditions.

The tidal river system is schematized as shown in figure 4. The network is composed of 25 branches (identified by roman numerals) that join or terminate at 25 junction locations (identified by numbered boxes). Junctions that do not constitute tributary or inlet locations in figure 4 were included in the network schematization to accommodate potential nodal flows (point source inflows or outflows such as sewage treatment outfalls or pump withdrawals) or to account for abrupt changes in channel characteristics.

A total of 66 cross sections were used to depict the channel geometry in 52 flow segments. Whereas the coefficient matrix of segment flow equations would require 15,376 computer words, use of branch-transformation equations reduces the matrix size to 10,000 words. The computational effort required to effect a solution is also proportionally reduced.

In the tidal Potomac River model, flow discharges derived at a rated gaging station (01–6465.00) 1.9 km upstream from Chain Bridge are used as boundary values at junction 1. Water-surface elevations recorded at a gaging station (01–6554.80) at Indian Head are used as the downstream boundary values at junction 19. All other external boundary conditions are fulfilled by specifying that zero discharge conditions prevail at the upstream tidal extent of the particular channel or embayment.

Water-surface elevations recorded near Key Bridge (station 01–6476.00), near Wilson Bridge (station 01–6525.88), and near Hains Point (station 01–6521.00) were used to calibrate and verify the model. (See figs. 3 and 4.) Model-computed discharges were also compared with discharges measured for complete tidal cycles at Daingerfield Island, Broad Creek, and Indian Head.

In the model calibration process, values of 0.6 and 0.5 were assigned to weighting factors θ and χ, respectively. Eta values in the calibrated model range between 0.0275 at Chain Bridge and 0.019 at Indian Head. A value of 1.06 was used for the momentum coefficient, β, and a 15-minute time step was used in the simulations. Using

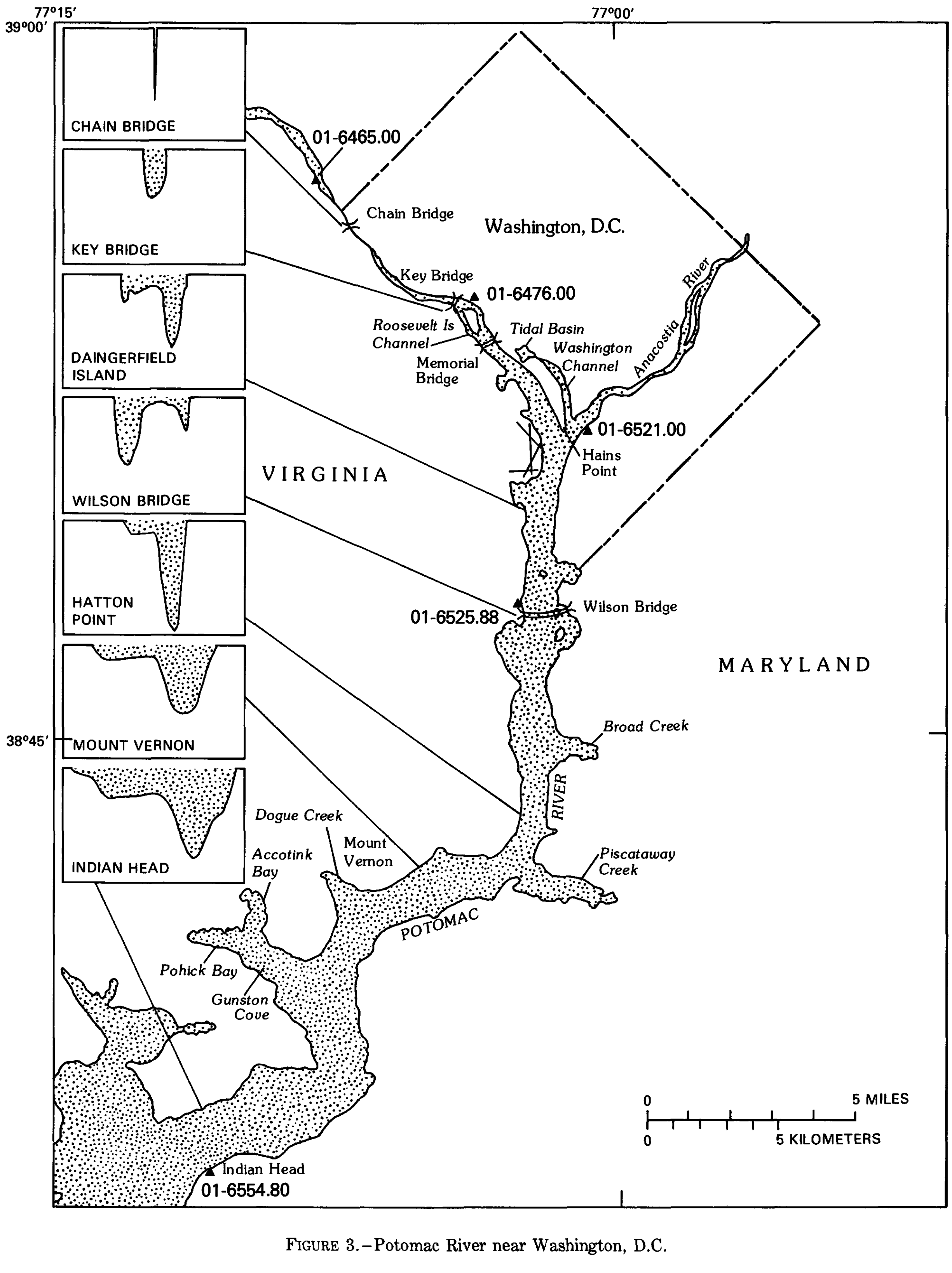

FIGURE 3.—Potomac River near Washington, D.C.

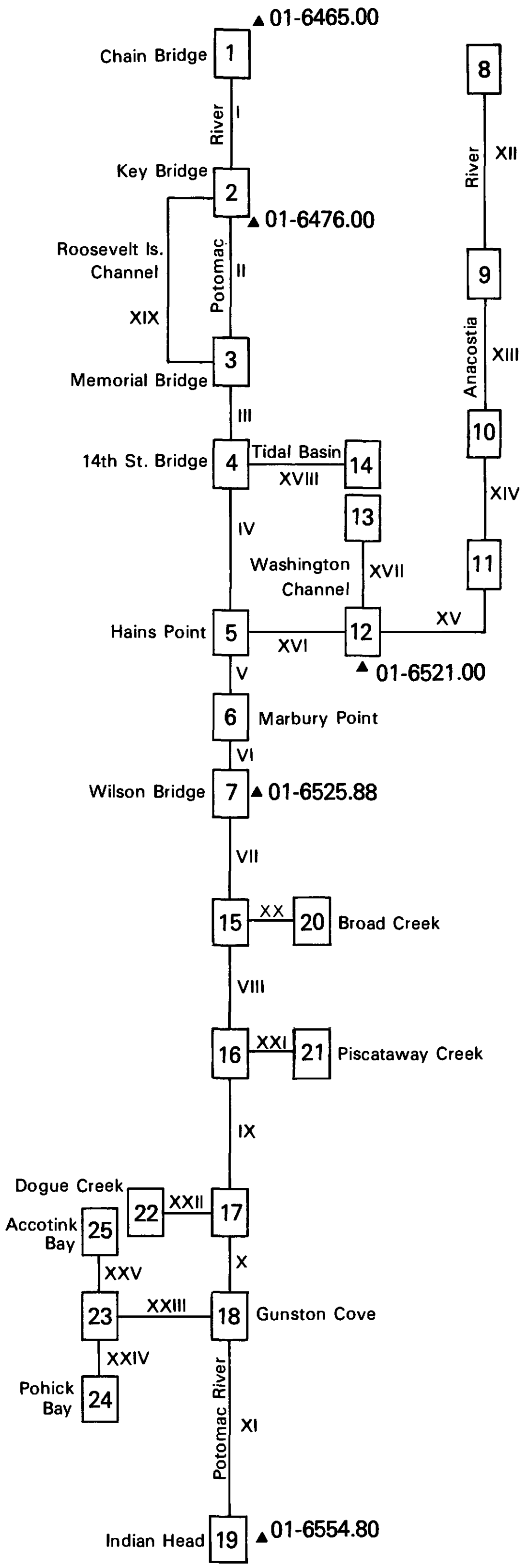

FIGURE 4.–Schematization of the tidal Potomac River system for the branch-network flow model.

these parameter assignments, the model has satisfied convergence criteria, set at 0.46 cm and 3.54 m^3/s for water-surface elevations and flow discharges, respectively, in fewer than two iterations per time step.

In figure 5, model-computed discharges are plotted against discharges measured at Indian Head from 2015 hours on June 3 to 0830 hours on June 4, 1981. This 15-hour simulation, from 1900 hours on June 3 to 1000 hours on June 4, required 10.3 CPU seconds on an Amdahl 470/V7 computer. On the average, 1.4 iterations were required per time step. As is evident from the plot, there is excellent agreement between computed and measured

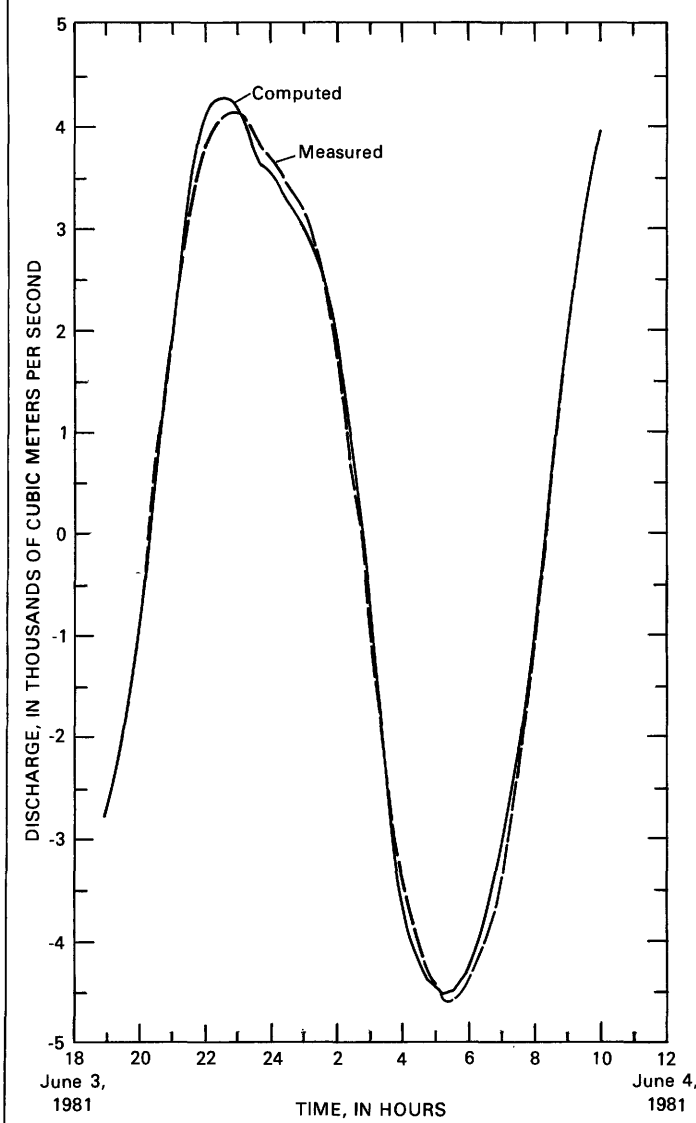

FIGURE 5.–Model-generated plot of computed versus measured discharges for the Potomac River at Indian Head, Md., on June 3-4, 1981.

discharges. Computed and measured ebb and flood volume fluxes compare within +0.6 and −2.3 percent, respectively.

This application of the model clearly demonstrates its adaptability to the simulation of unsteady flow in a network of interconnected channels.

MODEL USE

Mathematical/numerical models can address a variety of practical hydrologic field problems. They can be used, for example, to provide flow information for complex interdisciplinary riverine and estuarine investigations, to appraise hydraulic project-design alternatives, and to support environmental-impact assessments. Such varied uses emphasize the need, however, for models and (or) their supporting data base systems to provide efficient means of inputting and managing the required data and of analyzing and displaying computed flow information in a variety of graphical, pictorial, and alphanumeric forms.

To satisfy this need, the branch-network model has a wide range of graphical-display capabilities for computer generation of line-printer-drawn, mechanically drafted, or optically produced plots. These graphical capabilities not only help expedite model calibration and verification, but also provide a unique, rapid, and economical mechanism for portraying the flow and transport information required for various water-resources investigations. As an example, in figure 6 the particle-tracking capability of the model is illustrated in a model-derived plot. The time-of-travel graph of figure 6 depicts the movement of seven simultaneously injected index particles (labeled A through G) along the main Potomac River channel. The inflow-discharge hydrograph representing the upstream boundary condition is plotted above the time-of-travel graph. The stage hydrograph representing the downstream boundary condition is plotted below the time-of-travel graph. Model output such as this can be used to gain insight into the tidal-cycle variability in the concentration and dispersal of nutrients and sediments, as these are alternately or concurrently influenced by freshwater inflow conditions, meteorological effects, and tidal fluctuations.

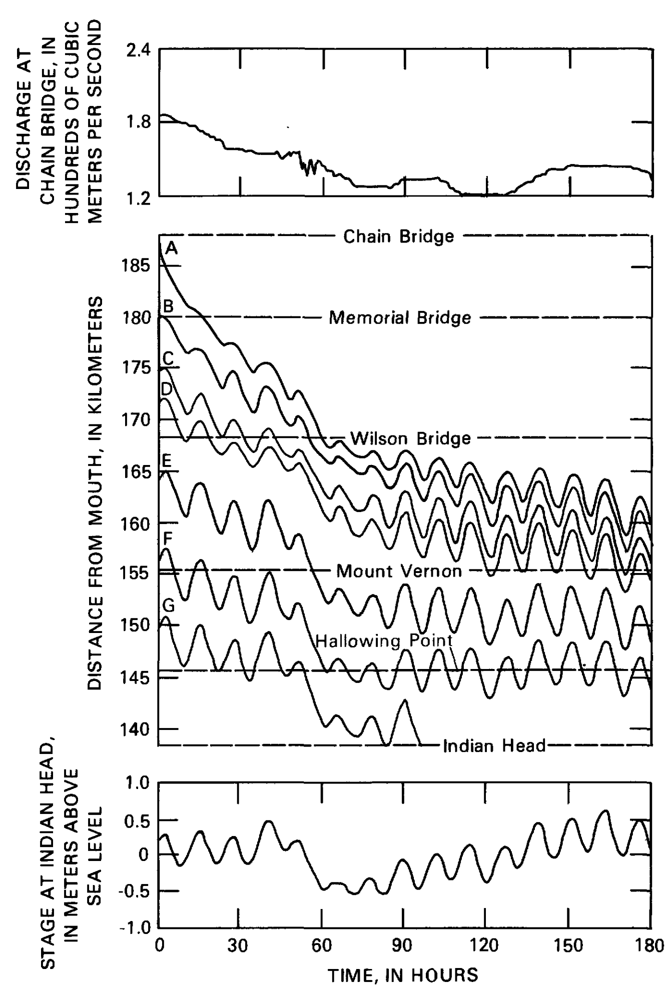

FIGURE 6.—Time-of-travel plot of injected particles for the Potomac River from midnight of November 30, 1980, to noon of December 8, 1980.

SUMMARY AND CONCLUSIONS

An operationally oriented, usable model has been developed to compute flow and transport information for a single open-channel reach or an interconnected network of open channels. The branch-network flow model, along with its supporting operational data systems (Schaffranek and Baltzer, 1978; Regan and Schaffranek, 1985), constitutes a complete one-dimensional numerical-simulation system. Based on a comprehensive set of unsteady flow equations and structured to accommodate a diversity of complex open-channel configurations, the model has a wide range of utility, as is exemplified by the numerous applications cited. Two specific applications are illustrated. The model includes numerous graphical display capabilities that provide both model engineers and water managers with flow information compiled and condensed into easily comprehensible formats tailored to suit their specific requirements. One specific output type—designed to depict the transport properties and flushing capacity of a riverine network—is illustrated herein; others have been reported elsewhere (Lai and others, 1978; Schaffranek and Baltzer, 1978; Lai and others, 1980; Schaffranek and others, 1981; Schaffranek, 1982).

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