Hydraulic Geometry of River Cross Sections— Theory of Minimum Variance

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SYMBOLS

b	hydraulic exponent of channel width, in the proportionality between width and water discharge, for example, $W \propto Q^b$
b_1	exponent of width included in $b_1/(b_1+f_1)$, as estimated from a width-versus-area plot.
d_{50}	grain size at which 50 percent of the distribution is finer (d_{10}=10 percent finer, and so forth.)
f	hydraulic exponent of mean water depth, in the proportionality between depth and water discharge
f_1	exponent of depth included in $b_1/(b_1+f_1)$, as estimated from a width-versus-area plot
ff	Darcy-Weisbach friction factor for wide channels, $=8gDS/V^2$
m	hydraulic exponent of mean velocity, in the proportionality between mean velocity and water discharge
n	resistance coefficient in Gauckler-Manning formula
r	correlation coefficient
s	distance
y	hydraulic exponent of friction factor, in the proportionality between friction factor and water discharge
z	hydraulic exponent of slope, in the proportionality between slope and water discharge
A	cross-sectional flow area
A_{max}	cross-sectional flow area at the upper end of the hydraulic geometry power relation
A_{min}	cross-sectional flow area at the lower end of the hydraulic geometry power relation
C	Chezy resistance coefficient
D	mean flow depth=A/W
D_{max}	mean flow depth at the upper end of the hydraulic geometry power relation
D_{min}	depth at the lower end of the hydraulic geometry power relation
Q	water discharge
R	hydraulic radius
S	slope or energy gradient
S_o	sorting coefficient=log d_{90}—log d_{10}
S.E.	standard error
V	mean flow velocity
W	water-surface width
W_{max}	water-surface width at the upper end of the hydraulic geometry power relation
W_{min}	water-surface width at the lower end of the hydraulic geometry power relation
X	horizontal distance from channel center toward bank
Y	depth at distance X
Y_o	maximum depth at channel center
Δ	difference between the logarithms of two quantities
σ	standard deviation
τ	bed shear stress
θ	angle of bank inclination
$\overline{\theta}$	average inclination of the two banks at a cross section

CONVERSION FACTORS

English	*Multiply by*	*Metric*
ft (feet)	0.305	m (meters)
ft (feet)	304.8	mm (millimeters)
ft^2 (square feet)	0.0929	m^2 (square meters)
ft^3 (cubic feet)	0.0283	m^3 (cubic meters)
mi^2 (square miles)	2.59	km^2 (square kilometers)

HYDRAULIC GEOMETRY OF RIVER CROSS SECTIONS—THEORY OF MINIMUM VARIANCE

By GARNETT P. WILLIAMS

ABSTRACT

This study deals with the rates at which mean velocity, mean depth, and water-surface width increase with water discharge at a cross section on an alluvial stream. Such relations often follow power laws, the exponents in which are called hydraulic exponents. The Langbein (1964) minimum variance theory is examined in regard to its validity and its ability to predict observed hydraulic exponents.

A major part of the study is devoted to identifying the important variables to use with the theory. These variables are velocity, depth, width, bed shear stress, friction factor, slope (energy gradient), and stream power. If the slope at a particular station is constant, only the first five of these variables need be considered.

The second aspect of the study tests the theory against field data. The 165 cross sections used reflect the following ranges of hydraulic exponents: $0.00 \leq b \leq 0.82$ (width), $0.10 \leq f \leq 0.78$ (depth), and $0.03 \leq m \leq 0.81$ (velocity). Flow conditions range from 0.000283 cubic meters per second (0.01 cubic feet per second) to 1,980 cubic meters per second (70,000 cubic feet per second), widths from 0.31 meter (1 foot) to 579 meters (1,900 feet), mean depths from 0.031 meter (0.1 foot) to 10.7 meters (35 feet), and median bed-material sizes from 0.06 millimeter to 100 millimeters. Most geographic regions of the contiguous United States are represented. The original theory was intended to produce only the average hydraulic exponents for a group of cross sections in a similar type of geologic or hydraulic environment. The present test shows that the theory does indeed predict these average exponents, with a reasonable degree of accuracy.

An attempt to forecast the exponents at any selected cross section was only moderately successful. Empirical equations are more accurate than the minimum variance, Gauckler-Manning, or Chezy methods. Predictions of the exponent of width are most reliable, the exponent of depth fair, and the exponent of mean velocity poor.

INTRODUCTION AND PROBLEM

Rivers have always been the arteries of civilizations. Because societies are so closely dependent upon the flow of water, people have for many years looked for methods of predicting the relations among the hydraulic features of a river—the water discharge, mean depth, width, mean velocity, and other variables. Such flow characteristics affect not only man but also the plants and animals living in or along the river.

The subject of this study is the rates at which water-surface width, mean depth, and mean velocity change with water discharge at a given cross section or station on a stream. Only streams that have loose particles on the bed will be considered. For such alluvial streams, there are no reliable ways to predict the rates of change of the flow variables mentioned above. Accurate methods are elusive because of the irregular shape of the cross section and the changing roughnesses of the flow boundary. The flowing water molds the loose particles into various patterns and configurations, and these roughness changes can vary with discharge. Bed roughness can also vary with distance across the stream. Under certain circumstances, such changes in bed roughness have been associated with abrupt changes in mean water depth and mean velocity. A search for general relationships can be complicated further by the variability of bank roughness, which differs considerably with lithology, vegetation, and other factors.

This paper begins with a discussion of basic relations and of the minimum variance theory (Langbein, 1964). The rest of the paper has three main parts. The first part of the study examines the question of which variables to use with the minimum-variance theory. The second, treated concurrently with the first, tests the theory in regard to its ability to predict the average rates (considering a large group of rivers and cross sections) at which mean velocity, depth, and water-surface width change with discharge. Finally, the third part is an attempt to find an objective way to forecast the hydraulic relations at any given cross section on an alluvial channel.

I would like to thank Walter B. Langbein, Edward J. Gilroy, Marshall E. Moss, and William H. Kirby for their many helpful discussions on certain as-

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BASIC VARIABLES AND RELATIONS

The multitude of variables that may be involved when water flows down an alluvial channel can be classified into flow properties (discharge, mean velocity, mean depth, etc.), water characteristics (temperature, specific weight, and viscosity), channel features (alinement or sinuosity, slope or energy gradient, shape of cross section), and sediment-related features (rates and grain sizes of sediment in transport, scour and fill, roughness of bed and banks, cohesiveness of bed and banks, sizes and shapes of boundary particles, and other aspects).

The flow variables of primary interest in this paper are the discharge or flow rate, Q; water-surface width, W; mean depth, D (defined as cross-sectional flow area A divided by W); and mean velocity V (defined as Q/A). The continuity relation specifies that $Q=VDW$.

Changes in these variables commonly are studied for two situations. The first is at a given station or cross section on a river, and the second is for a series of stations proceeding downstream on a river (Leopold and Maddock, 1953). Only the at-a-station case is treated in this paper.

A wealth of data (Leopold and Maddock, 1953; Stall and Yang, 1970; Fahnestock, 1963; and many others) shows that at cross sections on many rivers, canals, and laboratory flumes, the variables velocity, depth, width, slope S, and friction factor ff (defined later), all considered dependent, often change in proportion to some power of water discharge, considered independent. Thus

$$V \propto Q^m \quad (1)$$

$$D \propto Q^f \quad (2)$$

$$W \propto Q^b \quad (3)$$

$$S \propto Q^z \quad (4)$$

$$ff \propto Q^y \quad (5)$$

where the exponents, called hydraulic exponents, represent the rate of change of the dependent variables with change in Q. Power relations of the sort exemplified in equations 1–5 plot as straight lines on logarithmic paper. Leopold, Wolman, and Miller (1964, p. 215–281) discuss these hydraulic relations in some detail.

Power functions do not necessarily hold for the complete range of flows at a given cross section (Richards, 1976). Some of the many factors that can interrupt or prevent consistent relations over a wide range of discharge are discussed later. Also, more than one power relation can exist for within-bank flows. For instance, very low flows may follow one power law, and flows approaching bankfull may follow another. Finally, Richards (1973) points out that for some cross sections simple power functions do not apply. He suggests quadratic or higher order curves for such sections.

Hydraulic relations change drastically when a river overflows its banks. Such overbank flow will not be considered in this paper.

The following discussion deals with equations 1 through 5 and other equations of this form.

The general problem is to determine the hydraulic exponents wherever a power function relates the dependent and independent variables. Special attention will be devoted to the exponents of velocity, depth, and width (m, f, and b, respectively, in equations 1 through 3). This paper deals only with the exponents or rates of change. It does not consider methods of predicting absolute values of mean depth, width, and mean velocity.

Inserting the power equations 1 through 3 into the continuity relation $Q=VDW$ gives $Q^1 \propto Q^m Q^f Q^b$. The basic relation among exponents therefore, is $m+f+b=1.0$.

FACTORS THAT CAN INFLUENCE THE HYDRAULIC EXPONENTS

There is very little information on how water, channel, and sediment features affect hydraulic exponents. Intuitively, it can be reasoned that any water characteristic effects probably are small. For lack of evidence (admittedly not a proof of the assumption), these water features will be neglected in this analysis.

Of the channel features, channel alinement could be important because the cross-sectional flow pattern in a meander bend is different from that in a straight reach. Knighton (1975, p. 206), studying selected streams in England, found a lower rate of change of width on straight-reach sections than in meander sections. Channel alinement or sinuosity will largely be eliminated as a factor in this study by dealing only with stations located on straight or slightly curving reaches.

Water-surface slope, or energy gradient of the flow, is an important variable in many formulae for calculating discharge and mean velocity. A channel's slope is determined primarily by the general topography of the landscape, but the water-surface slope may vary somewhat with discharge at a station. In some parts of the present study, the slope at a station will be assumed constant, even though some error may thereby be introduced. The slope for the reach that includes the cross section will be studied in a later part of this investigation as a possibly important variable.

There may never be an accurate way to account for all channel-shape irregularities in alluvial channels. Some simplifications are unavoidable. Channel shape in this study is described by width/depth ratios, by the approximate average angle of inclination of the banks, and by certain other geometrical attributes of the cross section. These features are defined and examined later in this paper.

The hydraulic exponents for wide, flat channels (large values of width/depth) should differ from the exponents for deep, narrow ones. In a deep, narrow channel the water depth increases more rapidly with given increases in Q. The exponent of discharge associated with depth (hereafter called the exponent of depth, with symbol f) therefore should be higher in such channels, and m and b should be correspondingly lower.

The angle of inclination of the banks should also affect the exponents (Lewis, 1966; Knighton, 1974). If the banks are firm and vertical, the width remains constant with change in discharge, and the exponent of width (b in equation 3) is zero. At the other extreme, very flat banks would allow the width to increase considerably for a given increase in discharge, and the exponent of width would then be large.

The cohesiveness or the erodibility of the bed and bank material varies with the degree of consolidation of the particles, the grain sizes and their size-frequency distribution, the orientation, packing, and specific weight of the grains, the electrochemical bond between particles, the bulk density of the particles, antecedent moisture content, the age of the deposit (in many instances), and the water temperature (American Society of Civil Engineers, 1968; Partheniades and Paaswell, 1970; Fisk, 1952; Schumm, 1960; Raudkivi, 1967). These factors exert their influence in various ways, but their net result generally appears in the channel shape. For example, Friedkin (1945, p. 17) found that deep, narrow channels developed where banks were highly resistant to erosion. He noted that as bank resistance decreased the channels became progressively wider and shallower. Schumm (1960) plotted data for 69 rivers of the western United States and found that, for his data, the channel width/depth ratio was about inversely proportional to the percentage of silt and clay in the bed and banks. Wolman and Brush (1961) found that the force required to move the grains that make up the banks was a chief determinant of the channel shape. Knighton (1974) concluded that the bank silt-clay content is strongly correlated with bank inclination for 12 rivers in England. Thus, the way in which bank erodibility affects hydraulic exponents will be accounted for in this study mostly by an evaluation of the shape of the channel cross section.

Little is known as to whether the rate of sediment transport independently exerts an influence on the mean velocity, mean depth and width, and on their hydraulic exponents. Evaluation of any such effect is beyond the scope of this study. The sediment transport rate is generally associated with the channel shape. Channels that carry relatively large quantities of sediment, especially as bedload, tend to be wide and shallow. Those that carry small bed loads tend to be relatively narrow and deep. Consideration of the channel shape, therefore, may reflect any influence of the prevailing sediment transport rates on the hydraulic exponents. Also, the sizes of the moving particles help determine the kind of bed roughness for a given discharge.

The changeable bed roughness is associated with the mean flow depth. Because of this interrelation, an influence of bed roughness on hydraulic exponents cannot automatically be ruled out. Bed roughness depends on the sizes of the bed particles (grain roughness) and on the bedforms into which the flowing water molds these particles (form roughness).

The way in which the sizes of bed particles affect the hydraulic exponents is unknown. However, there is evidence (for example, Hack, 1957) that this factor may be important in stream behavior. Therefore, the sizes of the particles on the bed will be considered in this investigation.

The form roughness (ripples, dunes, and so forth) of alluvial channel beds often changes with discharge in a manner predictable only qualitatively. Some investigators believe that the change in water depth associated with the change in form roughness for certain limited flow and sediment conditions is a very important problem in alluvial streams. The hydraulic data plotted in this study cover many types of form roughness; however, the hydrographer rarely recorded the bedforms at the time of his discharge measurement. Therefore, this study will not analyze the role of changing bedform roughness on exponent values. The important point is that hydraulic exponents were readily definable, whatever the kinds of bed roughness. This suggests that if changes in depth did result from changes in bed-form roughness, such depth changes were not significant enough to disrupt the plotted power relations or could not be measured with sufficient accuracy.

A frustrating practical problem on some sand-bed streams is the "discontinuous" stage-discharge relation that sometimes occurs (Colby, 1960). For a given discharge, the stage or elevation of the water surface (referenced to a fixed bench mark) can vary from day to day because of the sediment moving through the reach. Development of a sand bar, for example, can cause the water level to rise and can disrupt the previous relation of water level to discharge. Many of the 165 stations examined in the present study have sandy beds. A number of these are notorious "shifters"—stations where the discharge associated with a given water elevation is not constant with time. These stations nevertheless had consistent relations between mean depth and discharge. Therefore, even though the relation between water level and discharge may change with time, the relation between mean depth and discharge may not be significantly affected.

Bank roughness could influence the exponents in that rough banks, for example, those with lots of vegetation, retard the water flow along the sides of the channel and, thereby influence the flow in the center. To eliminate this factor, the amount and kind of bank roughness should be constant for all stations. As this is impractical, some variation in observed exponents may be due to differences in bank roughness.

A number of features therefore could influence the hydraulic exponents. These relevant factors include width/depth ratios, average bank inclinations, other channel-shape aspects, channel slope, and the sizes and size distribution of bed particles. The channel-shape characteristics probably reflect the cohesiveness of the bank sediments and any influence of prevailing sediment-transport rates and types.

MINIMUM VARIANCE

THEORY

Data collected for this study show a wide range of possible values for each of the three main exponents (m, f, and b). What theories can explain the various observed exponents? Several different hypotheses would be desirable to insure greater impartiality when testing them against the data.

The literature contains at least seven theoretical or semitheoretical attempts to predict hydraulic exponents: the theory of minimum entropy production (Leopold and Langbein, 1962), the minimum-variance theory (Langbein, 1964), the minimal channel-mobility theory (Tou Kuo-Jen, 1964), the similarity principle (Engelund and Hansen, 1967), the minimum energy-degradation theory (Brebner and Wilson, 1967), the threshold-channel theory (Li, 1974), and the conservation-sediment transport theory (Smith, 1974). Only two of these—those by Langbein (1964) and Li (1974)—deal with the at-a-station case.

Li's theory is restricted to streams having gravel or boulder bed and banks in small watersheds (less than about 26 to 52 km^2 or 10 to 20 mi^2). For this limited situation, the exponents are fixed at $b=0.24$, $f=0.46$, and $m=0.30$, according to Li's theory.

Langbein's approach (Langbein, 1964, 1965; Scheidegger and Langbein, 1966) is flexible in regard to the predicted values of the hydraulic exponents and applies to a wide range of channel types. It applies to both the at-a-station and downstream cases. His theory takes a statistical or probabilistic viewpoint and tries to provide the average exponents for a group of stations of approximately comparable environments. As it is the only theory generally applicable to all at-a-station situations, Langbein's theory will be explored in detail in this paper. No new theories are introduced here.

The philosophy underlying a statistical or probabilitistic approach is that average or most probable relations are all that man can produce from theoretical considerations. This viewpoint holds that the particular complexities of any natural environment, such as the chance emplacement and distribution of rocks of various sizes on a streambed, are so numerous that the hydraulic exponents for any one spot can never be confidently and precisely calculated. For this reason, the minimum-variance theory was proposed as a method of forecasting the average relations for a group of rivers, with the understanding that any one case may not exactly agree with the expectation (see, for example, Langbein and Leopold, 1964). If such a hypothesis comes close to predicting observed relations, it may have the potential of becoming dependable enough to use in practical problems.

Many "laws" of science that have become established are based on the concept of the minimization of effects. Examples are Fermat's principle of light-ray travel, the principle of least action or least work (de Maupertuis or Le Chatelier, respectively), the principle of least constraint (Gauss), the principle of the straightest path (Hertz), the law of the equipartition of energy (Maxwell and others), the law of the survival of the fittest, and, in the business world, the law of supply and demand. The basic concept is that physical effects in the operations of nature, once having reached an equilibrium condition, change as little as possible from then on. In other words, a system tends to react to an imposed stress so as to minimize the disturbance, or to restore or to keep the previous conditions.

Consider the classical theorem of the equipartition of energy (Resnick and Halliday, 1960, p. 506–510). This principle deals with the various kinds of energy of a gaseous system—mainly kinetic energy of translation of individual molecules, kinetic energy of rotation of individual molecules, kinetic energy of vibration of the atoms in a molecule, and potential energy of vibration of the atoms in a molecule. These four kinds of energy represent different and independent ways in which the total energy of the system can be apportioned. An increase in the total energy could be absorbed in varying proportions by the four different kinds of energy. The theorem of the equipartition of energy states that the total available energy of a system containing a large number of molecules distributes itself in equal shares to each of the various ways in which the molecules can absorb energy.

The Langbein (1964) theory applies such a concept to the changes that occur in the variables of a river system. In keeping with the minimal principle, the changes in the variables are such that the total effect, action, work, or adjustment is a minimum. For a cross section on a river, the theory would suggest that all variables strive to resist any imposed change (maintain original equilibrium conditions), with the net result being that all of them change equally, insofar as possible. In other words, the dependent variables adjust by an equal percentage of their former values, subject to the restrictions of the situation. A typical restriction might be steep, cohesive banks that prevent the water-surface width from changing significantly as discharge increases.

A number of investigators have noted the applicability of such a minimization principle to alluvial channels. Examples are Velikanov (1947, p. 304), Mackin (1948, p. 492), Rubey (1952, p. 135), Leopold and Maddock (1953, p. 46), and Bretting (1958).

HYDRAULIC EXPONENTS BASED ON THE CONCEPT OF MINIMUM ADJUSTMENT

According to the minimum-variance theory just described, the problem is to find those particular hydraulic exponent values that, subject to any local physical restrictions, represent a minimum and equal adjustment to a change in the independent variable, usually water discharge. The thesis is that for these exponent values the sum of the squares of the exponents is a minimum. The logic behind this approach is best seen by analogies, presented below. Such a minimization approach is used in various branches of science and engineering to solve problems of indeterminate systems.

Consider a system with two dependent variables, say V and D. The question is how these variables will change in response to a new discharge. Select a discharge and plot the associated values of log V and log D on a graph (fig. 1) obtaining point x_1. (Power laws relate the variables, and log units are used here for the convenience of working with straight lines on arithmetic paper.) Now it rains upstream, and soon there is a new discharge at the station. This forces log V and log D to change by some magnitude, producing a new point x_2 on figure 1. The changes are $\Delta\log V$ and $\Delta\log D$. Assuming that neither depth nor velocity will decrease with the increase in discharge, point x_2 will lie somewhere in the quadrant that is bounded by a vertical line from x_1 (zero change in V, entire change absorbed by D) and a horizontal line from x_1 (zero change in D, entire change absorbed by V).

Given that the new Q causes the variables to move to some new point x_2, the total adjustment can be represented graphically by the straight-line distance s between x_1 and x_2. This distance, being the hypote-

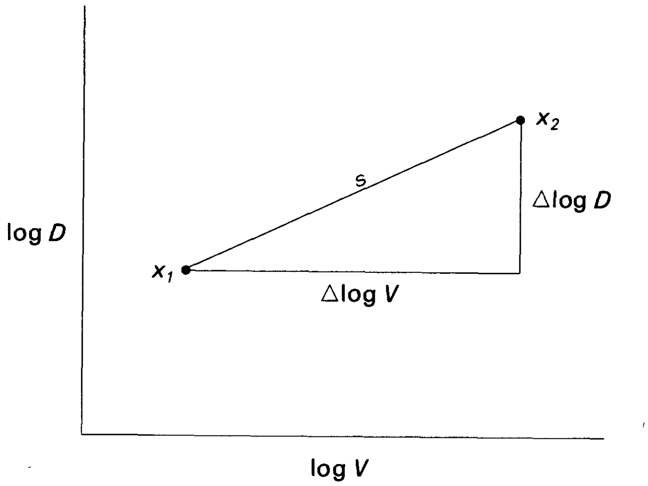

FIGURE 1.—Hypothetical change in two dependent variables, with resultant change represented by resultant or hypotenuse s; $s^2 = (\Delta\log D)^2 + (\Delta\log V)^2$.

nuse of a right triangle, is related to $\Delta\log V$ and $\Delta\log D$ by

$$s^2 = (\Delta\log V)^2 + (\Delta\log D)^2. \qquad (6)$$

The change in the independent variable, $\Delta\log Q$, is the same for both $\Delta\log V$ and $\Delta\log D$. Dividing all terms in equation 6 by the common constant $(\Delta\log Q)^2$ gives

$$\frac{s^2}{(\Delta\log Q)^2} = \frac{(\Delta\log V)^2}{(\Delta\log Q)^2} + \frac{(\Delta\log D)^2}{(\Delta\log Q)^2} = \left(\frac{\Delta\log V}{\Delta\log Q}\right)^2 + \left(\frac{\Delta\log D}{\Delta\log Q}\right)^2. \qquad (7)$$

This equation is, therefore, another way of expressing the amount of adjustment of the system.

The terms in parentheses on the right-hand side of equation 7 are hydraulic exponents. For example, from $V \propto Q^m$ we have

$$m = \frac{\Delta\log V}{\Delta\log Q}.$$

Equation 7 therefore can be written as

$$\frac{s^2}{(\Delta\log Q)^2} = m^2 + f^2. \qquad (8)$$

The left-hand side of this equation includes only the distance s and the constant $\Delta\log Q$ and therefore reflects s, the amount of adjustment. Thus, the magnitude of the adjustment, as indicated by the left-hand side of the equation, is proportional to the sum of the squares of the hydraulic exponents. Furthermore, least total adjustment occurs when s is a minimum, that is, when the sum of the squares of hydraulic exponents is the lowest number.

This type of relation can be extended to three or more dimensions to include any number of dependent variables. The least total adjustment for the entire system would be that for which the sum of squares of the hydraulic exponents is the lowest possible number, consistent with any local restrictions (for example, vertical rock banks) that may apply. Knighton (1977) derives this same principle from an entirely different viewpoint, namely Euclidean space.

Another analogy is the determination of the center of gravity of a two- or three-dimensional group of points. The center of gravity is that point from which the squared distances to all other points add up to the lowest number.

Examples that Langbein (Scheidegger and Langbein, 1966, p. C7) gave wherein the minimum sum of squares of changes corresponds to least total work are the displacement of joints in a truss and the distribution of the QH products (H being head loss) in a network of pipes.

The analogies thus far have suggested that minimizing the sum of the squares of the hydraulic exponents corresponds to least total change for the system as a whole. In addition, minimizing the sum of squares of exponents corresponds to an equal division of any change in an independent variable. Consider, for example, the analogy used in figure 1, whereby the entire increase in discharge was absorbed by V and D. The total adjustment (length of hypotenuse s of the triangle whose other sides are $\Delta\log D$ and $\Delta\log V$ in fig. 1) is greatest when the hypotenuse becomes equal to either $\Delta\log D$ or $\Delta\log V$, such that the other is zero. As $\Delta\log D$ becomes more equal to $\Delta\log V$, the hypotenuse (magnitude of adjustment) decreases, and it reaches its minimum value when $\Delta\log D = \Delta\log V$. In other words, the smallest net adjustment occurs when the dependent variables divide the imposed change equally amongst themselves.

A second example of this principle involves the many possible ways in which velocity, depth, and width can adjust to an increase in discharge. Consider the basic relation of the exponents, $m+f+b = 1.0$. For the hypothetical situation where no other variables are involved and where all three dependent variables (V, D, W) are unrestricted, the minimum sum of squares for all possible combinations of m, f, and b occurs when $m=f=b=1/3$. Mathematically, this sum of squares is $(.33)^2 + (.33)^2 + (.33)^2 = 0.3267$. Trial calculations show that any other values of m, f, and b, where $m+f+b=1.0$, produce a sum of squares that is greater than this minimum. For example, if $m=0.72$, $f=0.21$, and $b=0.07$, the sum of squares is 0.5674, which is greater than than

0.3267. Thus the variables absorb the change in Q equally when the squares of their exponents add up to the lowest number. Note also that the sum of squares of the exponents m, f, and b is a maximum (1.0000) when the change is wholly concentrated in one of the three dependent variables, such that the other two remain constant and their exponents are zero. A river would be least likely to adjust by changing in this manner, and it seems significant that the sum of squares of exponents is furthest from being a minimum for such a situation.

The concept of minimizing the sum of squares of changes or deviations is very similar to the well-known and widely accepted least-squares method for finding the best-fit relation to a group of values. The least-squares method can be used to approximate the "best-fit" relation not only to two variables or dimensions but also to three or more variables. Multiple regression and trend surface analysis are two examples.

What is the meaning of the term "variance" as used in Langbein's theory? Any group of water depths, mean velocities, and other values for a station can be analyzed statistically; for example, in regard to the arithmetic average of the depths, the standard deviation of those depth values, the variance (the square of the standard deviation), and so forth. The section "Statistical Variance and a Hydraulic Exponent" at the end of this report shows that if we take all the mean velocities measured for a station, list the logarithms of these velocities and compute the variance of this group, such a variance will be proportional to the square of the hydraulic exponent m (the exponent of velocity). The same is true of the other dependent variables and their respective exponents. Thus the square of a hydraulic exponent is proportional to the variance of the logs of the dependent variable, where variance is defined as the square of a standard deviation, as in normal statistical usage. Because of this close relationship and because the word "variance" is a convenient term which is already established in connection with the present theory, "variance" is used as a replacement for the more accurate phrase "square of hydraulic exponent." Thus, from $D \propto Q^f$ the square of the exponent (f^2) is called the variance of depth. The process of finding those exponents whose squares add up to the lowest possible number is termed "minimizing the variances." This concept of minimum variance is similar to, but not exactly the same as, the minimum variance of conventional statistics.

COMPUTATION OF MINIMUM VARIANCE

In calculating the most probable exponents in the hydraulic geometry relations, the simple laws of exponents apply. When two quantities are multiplied, their exponents are added; when one quantity is divided by another, the exponent of the latter is subtracted from that of the former. Shear stress in wide channels (τ), for example, is proportional to depth times slope. Substituting equations 2 and 4 into this expression, we have shear $\propto DS \propto Q^f Q^z \propto Q^{f+z}$. The variance of shear ("var shear") therefore is expressed by squaring the sum of the exponent of depth (f) and the exponent of slope (z), that is, var shear $= (f+z)^2$. The Darcy-Weisbach friction factor (ff) in wide channels equals $8gDS/V^2$, where g is acceleration due to gravity. This friction factor is proportional to depth times slope divided by the square of velocity, or $ff \propto Q^f Q^z / Q^{2m}$, and var $ff = (f+z-2m)^2$.

Because $m+f+b=1.0$, the variance of width, b^2, can be written in an alternate way as $(1-m-f)^2$. Similarly, $f=1-m-b$ and $m=1-b-f$. Also, one exponent may be known to have a certain relation to another, such as $b=0.25f$. In the latter case, the quantity $0.25f$ would be substituted for b, and the variances can all be written in terms of f. Finally, one exponent may be known in advance, permitting all variances to be written in terms of just one unknown. For instance, if $b=0.10$, then $m+f=0.90$ and $m=0.90-f$. Substituting 0.10 for b and $0.90-f$ for m allows all unknown variances to be written in terms of f. These many alternative ways of writing variances are used extensively to reduce the number of unknowns in the analysis.

Some factors may vary over a certain restricted range or are fixed so they do not change at all (remain constant). Such limitations are known as constraints. A constraint is merely a physical condition that governs the extent to which a variable can change. Examples are the fixed, constant width in flume experiments and the constant slope typical of certain reaches of some natural rivers and streams. The importance of a constraint is that it limits or otherwise influences the values that other variables can take. Constraints are usually present in nature, and in most cases they preclude a perfectly uniform distribution of any change in an independent variable. Partly for this reason, we rarely find a case where $m=f=b=0.33$.

One way of determining those particular values of the exponents that provide the minimum sum of variances is by trial and error. A case that Langbein (1965) used illustrates this method. Consider a river cross section where the banks are rigid and vertical so that the width is constant. The water-surface slope at some stream stations does not change significantly for a range of flow conditions (Leopold and Maddock, 1953, p. 36), so slope will also be considered constant for this example. Let the bed of the channel consist of unconsolidated sand, so that the channel resistance is adjustable. Now an increased water discharge arrives at the station. The question is how this and other increases in discharge will be reflected among the various pertinent dependent variables.

With the width constant, mean velocity and depth must absorb all of the new discharge. Changes in such factors as bed shear stress will also occur. Assume for illustrative purposes that this bed shear and the resistance to flow (friction factor), along with velocity and depth, are the only important dependent variables whose exponents are not already known. The problem is to find the values of the hydraulic exponents of these dependent variables.

The first step is to identify the variances of the dependent variables. In so doing, we must consider the constraints of constant width and constant slope. Because $Q=VDW$, or $Q^1=Q^mQ^fQ^0$ (width constant), this continuity expression specifies that $m+f=1.0$. The variance of velocity, m^2, can therefore be written in an alternate way as $(1-f)^2$. Shear in wide channels is ordinarily proportional to depth times slope or Q^fQ^z, so that the variance of shear would normally be $(f+z)^2$; however with slope constant its exponent $z=0$, shear stress varies only with depth, and the variance of shear is simply equal to f^2. The friction factor, proportional to DS/V^2, is proportional to D/V^2 or Q^f/Q^{2m} when slope is held constant. The variance in friction factor, being by definition the square of the exponent relationship, is, therefore, $(f-2m)^2$. Substituting $m=1-f$ in order to put this expression in terms of f, as the variances of the other variables are expressed, then gives the var *ff* as $(3f-2)^2$. The variances of the selected dependent variables, when width and slope are constant, are therefore

velocity	$(1-f)^2$
depth	f^2
shear	f^2
friction	$(3f-2)^2$

(For this initial example, it has been possible to write all variances in terms of one unknown, f. Although this will not be possible in many other cases, the alternative ways of writing variances should be used whenever possible in order to reduce the number of unknowns involved in the computation.)

The goal is that value of f for which the sum of all variances is a minimum. The theory holds that only under such a condition is the change in Q distributed as uniformly as possible among the dependent quantities. Suppose we guess that the exponent f—the rate of change in depth with change in discharge—is 0.3. We then compute the variances of each of the four dependent variables. Var vel, for example, is $m^2=(1-f)^2=(1.0-0.3)^2=0.49$. (See column 3 of table 1.) After the separate variances

TABLE 1.—*Trial and error example of finding* f *for which the sum of variances is a minimum*

Variable	Variance	Computed variance for value of f					
		0.3	0.4	0.5	0.6	0.7	0.58
Velocity	$(1-f)^2$	0.49	0.36	0.25	0.16	0.09	0.176
Depth	f^2	.09	.16	.25	.36	.49	.336
Shear	f^2	.09	.16	.25	.36	.49	.336
Friction	$(3f-2)^2$	1.21	.64	.25	.04	.02	.068
Sum of variances		1.88	1.32	1.00	.92	1.09	.916

are each computed in this manner, they are summed. Table 1 shows that when $f=0.3$, the sum of the variances is 1.88. Is this the minimum sum attainable? Continuing the trial and error method, the sum of variances turns out to be the lowest number when $f=0.58$. In other words, the change in Q is accommodated as equally as possible by the four major dependent variables when $f=0.58$, that is, when the rate of increase in log depth with increase in log Q is 0.58. The exponent of velocity, m, is therefore $m=1-f=0.42$. Width and slope are already known to be constant, so b and z both equal zero. Shear increases directly with depth, so the exponent of shear is 0.58 in this case. The rate of change in friction factor with increase in Q is equal to Q^f/Q^{2m} or $Q^{-0.26}$. The friction factor thus varies with the -0.26 power of discharge.

Such trial and error computations are laborious. Fortunately there is a quick, easy method of arriving at the answer $f=0.58$. The goal is that value of f that takes all of the variances, when added together, to a minimum. Therefore, add the variances figuratively and designate that their sum goes to a minimum:

$$\text{var vel}+\text{var depth}+\text{var shear}+\text{var friction}\rightarrow\text{minimum}$$

$$(1-f)^2+f^2+f^2+(3f-2)^2\rightarrow\text{minimum}$$

Squaring as indicated gives

$$(1-2f+f^2)+f^2+f^2+(9f^2-12f+4)\rightarrow\text{minimum}.$$

Collecting the like terms:

$$12f^2 - 14f + 5 \rightarrow \text{minimum.}$$

The value of f, representing the slope of the depth-discharge relation, is obtained by taking the first derivative. Because the function goes to a minimum, the derivative according to the rules of basic calculus is set equal to zero. Setting the first derivative equal to zero gives

$$24f - 14 = 0$$
$$24f = 14$$
$$f = 0.58.$$

(In fact, setting the first derivative equal to zero can give either a minimum or maximum value for a function, but in this case, the foregoing trial-and-error procedure has shown that $f = 0.58$ is the minimum value. Taking the first derivative for all other minimum variance computations also gives a minimum value, as can be verified by taking the second derivative. If the second derivative of a function is positive, then the first derivative has produced the minimum value. The second derivative of all minimum variance relations is alway positive.)

The summarized steps in the minimum variance computation are:

1. Determine an independent variable and write all other pertinent variables (dependent or constant) as power functions of the independent variable. Thus, if Q is independent, then $V \propto Q^m$, $D \propto Q^f$, slope $\propto Q^z$, shear $\propto DS \propto Q^f Q^z$, etc.
2. Define the variances (the square of the exponent) of all variables, using the same unknowns insofar as possible. Thus, for example, with $m + f + b = 1.0$ and with m and f necessarily involved, express var width as $(1 - m - f)^2$ rather than b^2.
3. Write the variances in a group and designate that the sum of the variances goes to a minimum.
4. Square any "compound" variances (a variance consisting of a sum of letters and possibly numbers) and collect like terms for the complete group.
5. Set the first derivative equal to zero and solve. If more than one unknown is present after step 4 is completed, set the derivative of each unknown equal to zero in turn while holding any other unknowns constant. Then solve the resulting equations simultaneously for the values of the unknowns.

Taking a more complex example, again presented only to demonstrate the method of computation, consider a reach of stream where width and slope are free to vary rather than being constant. The bed is deformable, as with the last example. Suppose a change in Q, as the independent variable, will be reflected in the dependent variables velocity, depth, width, bed shear, and frictional resistance. According to the minimum variance theory, what are the most probable hydraulic exponents in the power equations that relate these factors to discharge?

Each of the steps just listed is applied, in turn, to obtain the expected hydraulic exponents. Writing the variables as power functions of Q (step 1) and defining the variances (step 2):

Step 1 (relations)	*Step 2 (variances)*
$Q \propto Q^1$	1
$V \propto Q^m$	m^2
$D \propto Q^f$	f^2
$W \propto Q^b \propto Q^{1-m-f}$	$(1-m-f)^2$
$S \propto Q^z$	z^2
$\tau \propto DS \propto Q^f Q^z \propto Q^{f+z}$	$(f+z)^2$
$ff \propto DS/V^2 \propto Q^f Q^z / Q^{2m} \propto Q^{f+z-2m}$	$(f+z-2m)^2$

Writing the variances (of V, D, W, τ and ff for this example) as a group and designating this sum to be a minimum (step 3):

var vel + var depth + var width + var shear + var friction factor→minimum

$$m^2 + f^2 + (1-m-f)^2 + (f+z)^2 + (f+z-2m)^2 \rightarrow \text{minimum}$$

Squaring the compound variances and collecting like terms (step 4) gives

$$6m^2 + 4f^2 + 2z^2 + 1 - 2m - 2f - 2mf + 4fz - 4mz \rightarrow \text{minimum.}$$

Next the first derivative with respect to m is set equal to zero, holding f and z temporarily constant (step 5). This gives

$$12m - 2f - 4z - 2 = 0.$$

Then the first derivative with respect to f is set equal to zero, holding m and z constant:

$$-2m + 8f + 4z - 2 = 0.$$

Finally, holding m and f constant, setting the first derivative with respect to z equal to zero gives

$$-4m + 4f + 4z = 0.$$

In this manner, three equations are obtained (these last three, for this example) which are solved simultaneously for the three unknowns. For the present example, this solution produces $m = 0.14$,

$f=0.43$, and $z=0.29$. Since $m+f+b=1.0$, $b=0.43$. Velocity, therefore, varies as $Q^{0.14}$; mean flow depth, D, varies as $Q^{0.43}$, and so forth

INFLUENCE OF CHOICE OF VARIABLES

The present theory, like many statistical theories, unfortunately does not tell a person what variables to use in the computation. And the variables included in (or excluded from) the analysis have a strong effect on the values of the predicted exponents. For example, with width constant, velocity and depth absorb a change in Q; whereas, if width also takes up some of the change, then velocity and depth will change by lesser amounts, and their exponents will be smaller. This same principle applies to other variables such as shear stress and stream power. If such variables absorb some of the change in flow conditions, then the predicted exponents are affected. So a very important problem is the question of which variables to include in the analysis.

A simple example will show the extent to which the predicted exponents can vary with different sets of variables. Suppose the variables likely to be important are the discharge (independent), velocity, depth, friction factor, and shear stress. (All other factors constant or not influential.) The relations and variances are:

velocity $\propto Q^m$	m^2
depth $\propto Q^f$	f^2 or $(1-m)^2$
shear $\propto Q^fQ^0 \propto Q^f$	f^2 or $(1-m)^2$
friction $\propto Q^fQ^0/Q^{2m} \propto Q^{f-2m}$	$(f-2m)^2=9m^2-6m+1$

Table 2 shows the predicted exponents for all possible combinations of variables. As far as the grouping of variables is concerned, this table is not based on sound hydrologic reasoning nor on any theory—it merely lists in a systematic manner the different possible ways of combining two or more of the four dependent variables.

Each exponent in table 2 has a considerable range of values, depending on which particular set of variables is involved in the minimization. The range of m, for example, is from 0.30 to 1.0, and the exponent of the friction factor ranges from -2.00 to $+0.10$. Some values of exponents can be produced by more than one combination of variables (group nos. 1 and 2, 5 and 6, and 8 and 9), because slope is constant and hence shear stress varies directly with depth.

TABLE 2.—*Rate of change of dependent variables with increase in discharge, for different sets of variables*

Group No.	Dependent variables	Values of exponents: V Velocity (m)	D Depth (f)	τ Shear (f)	*ff* Friction factor ($f-2m$)
1	V, D	0.50	0.50	0.50	−0.50
2	V, τ	.50	.50	.50	−0.50
3	V, *ff*	.30	.70	.70	.10
4	D, τ	1.00	.00	.00	−2.00
5	D, *ff*	.40	.60	.60	− .20
6	τ, *ff*	.40	.60	.60	− .20
7	V, D, τ	.67	.33	.33	−1.00
8	V, D, *ff*	.36	.64	.64	− .08
9	V, τ, *ff*	.36	.64	.64	− .08
10	D, τ, *ff*	.45	.55	.55	− .35
11	V, D, τ, *ff*	.42	.58	.58	− .26

In view of the wide range of mathematically possible exponent values, how does one know what combination of variables to use in the minimum variance analysis?

In his original paper (1964), Langbein dealt with the components of stream power, QS, or velocity, depth, width, and slope as functions of discharge. Subsequently, (Langbein, 1965; Scheidegger and Langbein, 1966), for at-a-station cases, he added shear and friction factor to the group. He stated (1965, p. 304) that "there might be questions as to the proper variables, but these are used for consistency in the several examples." In the 1966 paper (p. C8), he suggested that a large set of problems can be explained by using as dominant factors the width, hydraulic radius, velocity, shear, and friction factor. Actually, depth was used in place of hydraulic radius in all the examples given. Langbein (1965) chose the combination of V, D, W, τ, and *ff* for at-a-station situations because minimizing the variances of these variables produced exponents closest to those of the few case samples he cited.

In both the 1965 and 1966 papers, Langbein treated the downstream cases—rivers and straight canals—as special or different from the at-a-station situation. For downstream cases, he minimized the variances of five different aspects of stream power.

The problem of selecting the correct combination of variables is very similar to that encountered in dimensional analysis, where you have a list of dimensionless terms and must decide which terms are insignificant. The variables used in earlier pages of this paper (shear, friction factor, and so forth) may or may not be the most important variables, and, unfortunately, the concept of least total adjustment by itself cannot suggest the most important factors. Two possible solutions are (1) introduce another theory to show which variables should be involved, and (or) (2) compare actual field data to the exponents predicted by various combinations of variables. Any "proof" of the minimum variance theory, in fact, must include the latter sort of comparison, and it is the second approach that will be followed in this paper. If the predictions for a given group of variables match the field observations, the correct or best combination of variables has been found and the theory acquires a certain degree of reliability. On the other hand, if no combination of variables produces exponents close to the field data, the minimum-variance theory could then be said to lack a firm factual basis.

An alternative to dealing with a standard set of variables is to use different combinations of variables for various hydraulic situations (Maddock, 1969). The difficulty with this procedure is knowing which variables to use for any given situation (Dozier, 1976).

TEST OF MINIMUM-VARIANCE THEORY WITH FIELD DATA

COLLECTION OF DATA

The U.S. Geological Survey has for many years measured the water discharge at selected stream stations throughout the United States. In making such measurements, the hydrographer obtains data that yield *V, D, W, Q,* and the shape of the channel cross section. Records of this sort afforded an opportunity to test the theory against actual data. Although the Geological Survey monitors thousands of stations, the selection criteria dictated by the present study eliminated the vast majority of sites. These criteria were that the stream have (a) a movable bed (silt, sand, gravel, cobbles, and (or) boulders), (b) no artificial or natural control or apron on the bed at the measuring site, (c) negligible influence on flow variables from bridge piers, (d) no history of wide-scale dredging, (e) no dam immediately upstream or any other feature that caused observable net degradation or aggradation along the gaging station reach, (f) no extremely heavy bank vegetation significantly affecting the flow range interest, and (g) a range of discharge preferably encompassing at least one log cycle over which the plotted hydraulic data showed well-defined power relations.

When making a discharge measurement, the hydrographer commonly walks as much as several hundred feet upstream or downstream from the gage to find a cross section that is easy to wade. The records for a given station therefore, often consist of measurements made at many cross sections along a reach. Unless the channel shape remains approximately constant along the entire reach, the plots will show a certain amount of scatter attributable to the variety of measuring sites (Wolman, 1955, p. 11; Lewis, 1965, p. 12–13). It was therefore necessary to inspect the original streamflow measurement notes and, for each gaging station, to select only data taken at the same cross section. In almost all cases, this task was accomplished by accepting data taken either at the same wading site or from cableways, as recorded in the hydrographer's field notes. In rare instances, the channel shape was sufficiently constant along the reach to permit the use of data taken anywhere along the reach. The requirement that measurements be made at the same section eliminated many of the stations that had passed the seven criteria listed in the previous paragraph.

Table 11 (appended to the end of this report) lists the 165 stations finally accepted for analysis. For each of these stations, I plotted values of *W, D,* and *V* versus *Q* on log paper and measured the hydraulic exponents graphically.

The 165 stations were chosen to represent many physiographic regions. Included are streams in different climates, soils, lithologies, and types of landscape; streams with beds of very small grains and streams with large cobblestones and even boulders on the bed; streams a few feet wide and streams many hundred feet wide, with a correspondingly wide range of typical discharges, depths, and velocities; and streams with banks ranging from firm and steep to rather flat and easily erodible. Some of the selected stations are on ephemeral reaches.

The observed exponents range from 0.00 to 0.82 for *b*, 0.10 to 0.78 for *f*, and 0.03 to 0.81 for *m*. The channel widths, or rather the values on the hydraulic-geometry plots, range from 0.31 m (1 ft) (minimum width on *W* versus *Q* relation) to a maximum of about 579 m (1,900 ft). Depths range from about 0.031 m (0.1 ft) to 10.7 m (35 ft). The smallest bed-material size (median diameter) is 0.06 mm, and the largest is about 100 mm (table 11). The lowest discharge on a hydraulic-geometry power relation is 0.000283 m^3/s (0.01 ft^3/s) (Belle Fourche River below Moorcroft, Wyo.) and the highest is about 1,980 m^3/s (70,000 ft^3/s) (Skagit River near Mt. Vernon, Wash.)

The period of time covered by the various discharges on any one plot averaged about 3 to 5 yr and ranged from about 1 to 17 yr, depending mainly on how often the more extreme discharges flowed and on how frequently the different flows were measured.

SOURCES OF ERROR

The basic data (*Q*, *V*, *D*, and *W*) for every station involve a certain amount of measurement error that could affect the hydraulic exponents. A discharge measurement is made by observing the total depth and the velocity at one or more intermediate depths, at each of many successive verticals across a stream (Buchanan and Somers, 1969). The mean velocity and an applied cross-sectional flow area at each vertical give the discharge for a subsection, and the various subsection discharges are summed over the entire stream width to get the total discharge. Carter and Anderson (1963) analyzed and discussed in detail some of the sources of error in such measurements. They concluded that with normal stream-gaging procedures, the errors due to the instrument and the general current-meter method would be about ± 2 percent or less in most cases and would be randomly distributed. However, in natural streams, such features as water waves and bed forms, especially in shallow depths and at extremely high and low discharges, add to the errors involved. The hydrographer estimates the accuracy of each discharge measurement by rating it excellent (2 percent error), good (5 percent), fair (8 percent), or poor (over 8 percent), according to the flow-, channel-, and instrument characteristics at the time of the measurement. Nearly all of the data in the present study were rated good or fair. The error in the plotted *Q*-, *V*-, and *D*-values, in other words, probably is no more than about 8 percent in most cases. Water-surface widths could be measured quite accurately and should have a negligible error for the present purposes.

Trial calculations with typical best-fit lines indicated that errors of 8 percent in the measurement of *Q*, *V*, and *D* could cause a difference of about 20 percent in the exponents of velocity and depth if many errors at each end of the best-fit line happened to be distributed so as to cause the maximum possible deviation in slope of the line. Similarly, the exponent of width could have a maximum of about 5 percent error. However, measurements of *Q*, *V*, *D*, and *W* have an equal chance of being off on either side of their true value, and, therefore, such measurement errors should tend to offset one another over a number of observations. Hence, any errors in the exponent values due to errors in measuring *Q*, *V*, *D*, and *W* probably are not significant.

The number of points on any one plot ranged from 10 to 105 and averaged 30 per graph for the entire study.

The lines on the plots of velocity, depth, and width versus discharge were, in most cases, fitted by eye. Those for stations 15, 41, 48, and 69 were fitted by least squares.

Figures 2 and 3 show the plots for two of the stations. These examples were selected because they represent approximately the least scatter (fig. 2) and the most scatter (fig. 3) for the 165 stations studied.

The amount of scatter on all 495 graphs (3 graphs for each of the 165 stations) was measured and is expressed in table 11 as an approximate percentage of the best-fit-line value of the dependent variable for any given discharge. The percentages encompass about 90 percent of the total number of plotted points for each station and were determined as follows.

On each hydraulic-geometry graph, two lines parallel to the best-fit line were drawn. One line excluded the 5 percent of the points having the greatest positive or upward departure from the best-fit line, and the lower line excluded the 5 percent of the points having the greatest downward departure from the best-fit relation. These two parallel lines thus included about 90 percent of the total number of plotted points. (This figure was selected in order to exclude the occasional outliers.) For any given discharge, the value of the dependent variable, as indicated by the upper line, was read and expressed as the percentage of the best-fit-line value by which it exceeded the latter. The corresponding percentage for the lower line is the percent of the best-fit line value by which the lower line falls below the best-fit line. These two percentages are both listed for each graph. They indicate the approximate range of percentage within which nearly all (that is, about 90 percent) of the plotted points fell, relative to the best-fit line. For example, a listing of 82/51 for velocity means that 90 percent of the plotted points fall between two parallel lines that are, respectively, 82 percent above and 51 percent below the best-fit line, at any given discharge. Incidentally, the popular correlation coefficient proved unsuitable for the present purpose because it varied with the slope of the line and the extent of the discharge range, as well as with the scatter. A line with a low slope gave a low

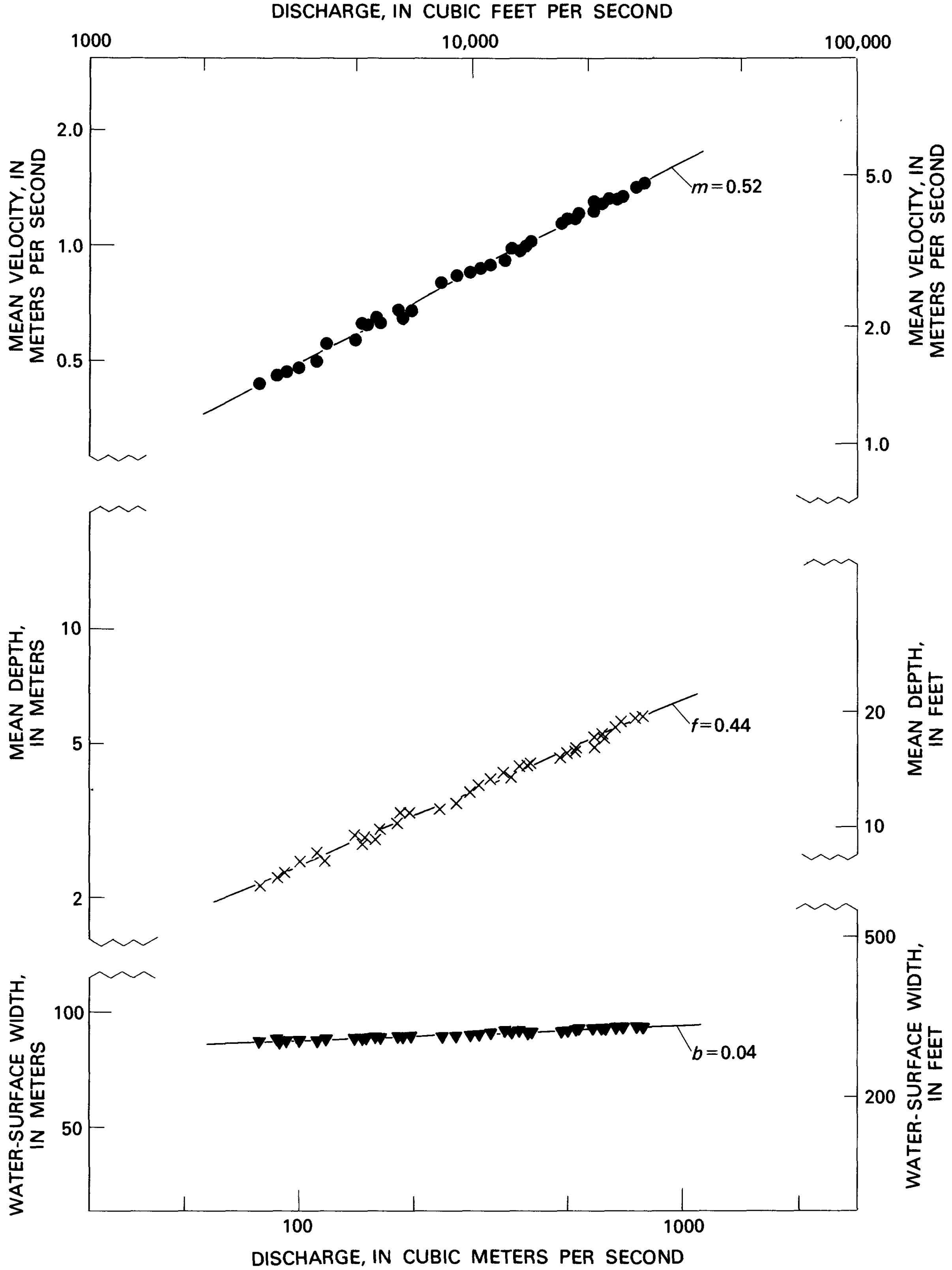

FIGURE 2.—Hydraulic-geometry plots for Colorado River near Grand Canyon, Ariz., 1970 (station 124, this study).

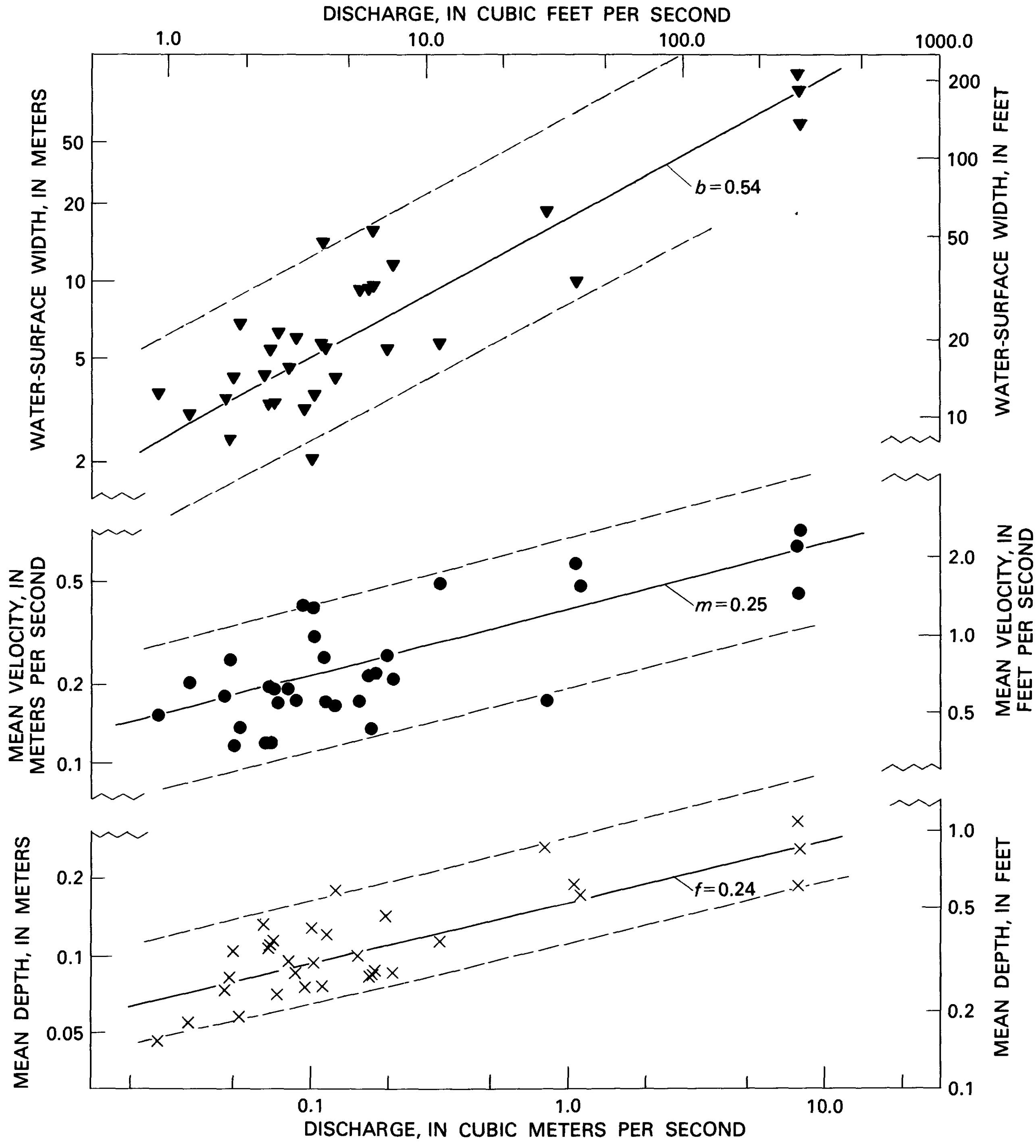

FIGURE 3.—Hydraulic-geometry plots for Prairie Dog Town Fork Red River near Childress, Tex. (station 60, this study). Dashed lines include about 90 percent of the plotted points.

correlation coefficient, which presumably implies a poor relation, even though all of the plotted points fell virtually right on the best-fit line.

The scatter on the log-velocity versus log-discharge graphs ranged from 2 to 110 percent with an arithmetic mean of 23 percent. For depth versus discharge, the range was from 2 to 120 percent with a mean of 24 percent. And for width versus discharge, from 1 to 110 percent with a mean of 21 percent.

The plots shown in figure 2 have scatter of about 3/4 percent for velocity, 5/4 percent for depth and

2/2 percent for width. On figure 3, the scatter is about 87/50 percent for velocity, 78/34 percent for depth, and 110/53 percent for width.

The final source of analysis error is the error due to fitting the lines by eye rather than by least squares (assuming least squares would be the best possible way). To evaluate this error, the exponents for 10 randomly selected stations were computed by least squares and compared to the corresponding eye-drawn exponents, the latter having been determined first in all cases.

For the 30 test plots (10 stations) examined, the maximum discrepancy between an exponent determined by least squares versus one fitted by eye was ±0.03 exponent units (table 3). The average absolute error or discrepancy, computed without regard to the sign, was about 0.01 exponent units for each of the three exponents. Drawing lines of best fit by eye, therefore, was reasonably accurate and reliable.

TABLE 3.—*Comparison of exponents determined by least squares with those fitted by eye*

Station No.	Least-squares exponents			Eye-fitted exponents			Discrepancy		
	m	*f*	*b*	*m*	*f*	*b*	*m*	*f*	*b*
1 --	0.51	0.42	0.07	0.51	0.43	0.07	0.00	0.01	0.00
16 --	.34	.59	.07	.35	.60	.08	.01	.01	.01
31 --	.52	.49	.01	.52	.48	.01	.00	—.01	.00
47 --	.36	.42	.22	.36	.41	.21	.00	—.01	—.01
64 --	.42	.53	.06	.40	.56	.05	—.02	.03	—.01
81 --	.20	.16	.67	.18	.15	.65	.02	.01	.02
98 --	.25	.14	.60	.25	.13	.61	.00	—.01	.01
115 --	.48	.50	.04	.47	.50	.02	—.01	.00	—.02
132 --	.35	.26	.39	.37	.23	.40	.02	—.03	.01
149 --	.71	.24	.05	.72	.24	.04	.01	.00	—.01
Average absolute error	---	---	---	---	---	---	.009	.012	.010

COMPARISON OF MEASURED TO THEORETICAL HYDRAULIC EXPONENTS

A number of variables, as discussed earlier, might have some influence on the hydraulic exponents. However, Langbein's papers suggest that most such factors cannot be taken into account individually in a minimum-variance analysis because their effects usually cannot be determined separately. He believes that in spite of the interaction and net influence of such variables, there will result in nature a statistical array of exponent vaues in which certain values (the averages) are more common. These most common values of *m*, *f*, and *b* represent a central tendency, and the correct combination of variables is that for which the minimization of variances yields the most common exponents.

Five cases, including four that Langbein (1965) specified, are tested here. The first case deals with stations where the water-surface width is approximately constant. The banks are firm and are steep enough to prevent the width from changing significantly with discharge. This category was arbitrarily defined as including all stations for which $b \leq 0.03$. These stations, a total of 22, are labeled "A" in table 11. For the second case, the banks again are firm but, in this case, they are not steep enough to keep the water-surface width approximately constant (76 stations). In table 11, these stations are labeled "B". Also, a few are labeled "R" for rock banks, or B/R for one rock bank and one firm-sediment bank. The third case (labeled "C") includes channels where the entire flow boundary is loose and easily eroded. The present study has 16 such stations. A fourth case (B/C, C/R) consists of the 51 stations having one firm bank and one noncohesive bank. Langbein (1965) did not examine this case.

In all four of the above cases, the energy gradient, or water-surface slope, is specified to remain essentially constant as discharge varies. (On some reaches, this may be only approximately true and may account for some unknown amount of discrepancy between predicted and observed results.)

The fifth case (D) consists of stations where the entire flow boundary is loose and readily erodible and the water-surface slope varies with discharge. None of the present stations were measured for slope changes and are here considered to have virtually constant slopes. However, some flume data from other studies are available for case D.

Classification of general bank firmness for each station was done subjectively from U.S. Geological Survey unpublished descriptions of the channel and banks at each station and from verbal descriptions given by Geological Survey hydrographers.

It is first necessary to find out whether a central tendency of exponent values exists for each of these cases. The few data for case D are insufficient to determine any clustering or distribution trend. For the other four cases, figure 4 summarizes the field data collected. The exponents for most cases have quite a wide range of values. However, with the possible exception of the exponent *b* for B/C stations, the distribution of exponent values for a given case and exponent does show an observable peak. Thus, a central tendency does exist for each exponent.

Some of the distributions in figure 4 are not symmetrical. However, plotting the data on both log-probability and arithmetic-probability paper showed that the distributions tend to be much more sym-

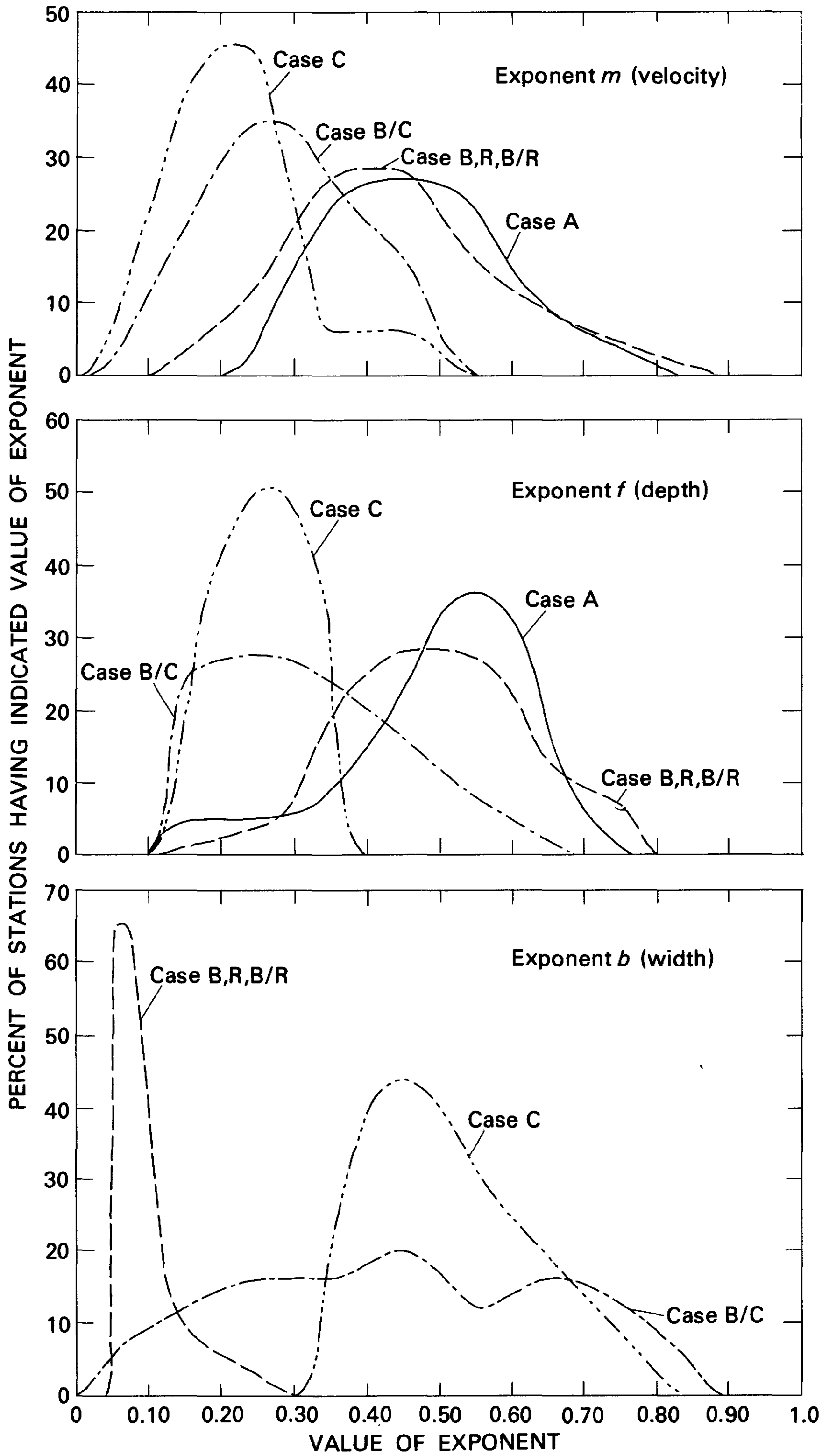

FIGURE 4.—Frequency distribution of exponent values, for different categories of stations. Case A: banks firm, width essentially constant. Case B, R, B/R: banks firm, width variable. Case C: banks noncohesive. Case B/C: one bank firm, the other noncohesive.

metrical on arithmetic than on log scales. (The only exception was case B, B/R, and R for the exponent b, where all b-values < 0.04 were arbitrarily eliminated beforehand for use in case A.)

In practice, the average exponent for a number of field stations may not represent the true central tendency, depending on the number of stations for which data have been acquired and on how close to the "ideal" case these stations are. As a first approximation, however, the average of a given exponent for the stations of each category should be reasonably representative. Two averages—the arithmetic mean and the mode—were used for all cases.

The next questions are, can the minimum-variance theory predict these average or most common exponents and, if so, what combination of variables provides those average values?

Case A

The simplest at-a-station situation consists of cross sections at which both width and slope are approximately constant. As far as V, D, and W are concerned, any change in discharge is absorbed only by V and D. The exponent b, therefore, is approximately zero (taken as ≤ 0.03 for present purposes), and $m+f \approx 1.0$. The field data (table 11) have means of $f=0.52$ (modal $f=0.55$) and $m=0.47$ (modal $m=0.46$) for the 22 applicable stations. Standard deviations are 0.13 and 0.13 exponent units for m and f. What group of variables, if any, predicts these average exponents in a minimum-variance computation?

Table 4 lists the theoretical exponents produced by minimizing the variances of various combinations of variables. The variables included in this table (V, D, τ, ff and VS, with width and slope constant) are those which can vary and which are deemed most likely to be important—a subjective but unavoidable approach. (QS and QS/W, proportional to stream power and stream power per unit bed area, respectively, may also be important and will be considered later. For this case these two variables both vary directly with Q and do not influence the computed exponents.) VS is proportional to stream power per unit weight of water. Yang (1972) believes this variable governs the nature of a stream network, the formation and behavior of meanders, the river profile, the formation of riffles and pools, and the rates of sediment transport.

TABLE 4.—*Theoretical rates of change of dependent quantities with increase in discharge, for different sets of variables*

[Case A: width and slope constant. Average field values: $m=0.47$ (mode 0.46); $f=0.52$ (mode 0.55)]

Number	Dependent variables	Value of exponents: V Velocity (m)	D Depth (f)	τ Shear (f)	ff Resistance ($f-2m$)	VS Power per unit weight (m)
1	V, D	0.50	0.50	0.50	—0.50	0.50
2	V, D, τ	.67	.33	.33	—1.00	.67
3	V, D, ff	.36	.64	.64	— .08	.36
4	V, D, VS	.33	.67	.67	,00	.33
5	V, D, τ, ff	.43	.57	.57	— .29	.43
6	V, D, τ, VS	.50	.50	.50	— .50	.50
7	V, D, ff, VS	.33	.67	.67	.00	.33
8	V, D, τ, ff, VS	.38	.62	.62	— .14	.38

Inspection of the predicted exponents shows an expected range of values for each exponent. No combination in table 4 exactly corresponds to the field averages of $m=0.47$ or 0.46 and $f=0.52$ or 0.55. Three groups (numbers 1, 5, and 6 in the table) produce exponents rather close to these values: (a) V and D, the minimization of which yields $m=0.50$ and $f=0.50$; (b) V, D, τ and ff, the minimization of which yields $m=0.43$ and $f=0.57$; and (c) V, D, τ and VS, for which $m=0.50$ and $f=0.50$. The evidence for stations having constant width and slope therefore suggests that if the minimum-variance theory is valid, one of the three combinations just listed may be the basic or best group of variables. For more general cases, these groups could also include one or more of W, S, QS, and QS/W, factors that are constant or noninfluential for case A. In fact, since the width W always absorbs some of a change in discharge (except, of course, where the banks are firm and vertical), width must be included as an indispensable variable. Thus, if minimizing the variances of one group of variables applies to all at-a-station situations, field data for case A narrow the choice to three general groups: (a) V, D, and W, possibly with S, QS, and (or) QS/W; (b) V, D, W, τ, and ff, possibly with S, QS, and (or) QS/W; or (c) V, D, W, τ, and VS, possibly with S, QS and (or) QS/W.

Case B

Case B consists of stations with cohesive but nonvertical banks with the water-surface slope again constant. The water-surface width, in other words, increases with increase in discharge but is controlled or constrained by the firm banks. The rate of change of width depends, at least in part, on the shape of the cross section or angle of the banks at various flow stages. Because of this control which the firm banks exert on the exponent of width b, Langbein (1964) felt that some relationship reflecting such control should be incorporated into the minimum-variance analysis. For this case, he introduced the constraint $b=0.55f$, saying this relation obtains for the stable channel described by one of Nizery and

Braudeau's (1955) sine equations. (See also Chow, 1959, p. 178, eq. 7–12). The equation supposedly describes a stable hydraulic section, such as the cross section of an erodible channel (canal) in which no erosion will occur at a minimum water area for a given discharge. The formula is

$$Y = Y_o \cos\left(\left[\frac{\tan\theta}{Y_o}\right]X\right) \qquad (9)$$

where Y = the depth at a horizontal distance X from the channel center, Y_o = the maximum depth at the channel center, and θ is the angle between the horizontal and the bank at bankfull stage and is taken to be the angle of repose of sand grains (about 33°).

For such a channel, the relation $b=0.55f$ results from taking the bankfull-stage cross section as given by the above equation, choosing various water-surface widths for different stages, getting the cross-sectional flow area corresponding to each width, computing the mean depth associated with each water-surface width and flow area, and finally plotting water-surface width (on the ordinate) versus mean depth on logarithmic paper. The points thus plotted follow a power law and have a slope of 0.55, which means $W \propto D^{0.55}$. Thus, $b/f=0.55$ or $b=0.55f$.

Kennedy, Richardson, and Sutera (1964, pp. 338–339) legitimately asked whether the above cosine equation—a theoretical one intended for canals—does, in fact, describe the shape of river channels having movable beds and relatively firm banks. Thus, for testing the minimum-variance theory, an important question centers on the validity of Langbein's assumed relation $b=0.55f$. Data accumulated for the present study should be sufficient to determine whether this relation is accurate.

Table 11 shows 76 stations labeled B, R, or B/R; however, two of these (stations 20 and 25) possibly should be classified as having at least one "loose" boundary. To be safe, these two stations were eliminated for this minimum variance analysis, leaving 74 stations for testing case B.

Of these 74 stations, the value $b/f=0.55$ is approximated or exceeded in only three instances. For the vast majority of stations, b is a relatively low percentage of f. The average value of b/f for all 74 stations is 0.19, with two-thirds of the cases falling within the range $0.09 \leqq (b/f) \leqq 0.28$. The average value of b/f, therefore, is reasonably well defined, and the assumption $b/f=0.55$ definitely is not justified for the stations studied here.

In keeping with the policy of dealing with average values, case B was tested using the contraint that $b=0.19f$. This empirical relation presumably accounts for the control that the firm banks exert on the hydraulic exponents. However, a plot (not shown) of b versus f for the 74 stations shows that b tends to decrease with increase in f, rather than increasing as the equation $b=0.19f$ suggests. An eye-fitted line on the graph yields the very approximate relation $b=0.12-0.06f$. This may express the relation between b and f as well as the former equation and was used in an alternate test of case B. As a matter of fact, the value of b for all 74 firm-bank stations tends to be low. The average is 0.08, and two-thirds of the cases fall within the range $0.04 \leqq b \leqq 0.11$. It is, therefore, not unreasonable to take $b=0.08$ as an average, for these 74 stations. A third test of case B, accordingly, was made using the constraint that $b=0.08$, so that $f+m=0.92$.

Table 5 shows the exponent values predicted by minimizing the variances of those combinations of variables that survived case A. The variables S and QS are constant and noninfluential, respectively, for case B; for brevity, they are not included in the

TABLE 5.—*Theoretical rates of change of dependent factors with increase in discharge for groups of variables surviving case A and for different constraints*

[Case B: firm banks. Average field values: $m=0.42$ (mode 0.40); $f=0.50$ (mode 0.48); $b=0.08$ (mode 0.07), for 74 stations]

Sets of dependent variables	Constraint	Values of exponents						
		V Velocity (m)	D Depth (f)	W Width (b)	τ Shear (f)	ff Friction factor ($f-2m$)	QS/W Power per unit area ($1-b$)	VS Power per unit weight (m)
V, D, W	$b=0.19f$	0.42	0.49	0.09	0.49	—0.35	0.91	0.42
$V, D, W, QS/W$		.34	.55	.11	.55	— .13	.89	.34
V, D, W, τ, ff		.36	.53	.11	.53	— .19	.89	.36
$V, D, W, \tau, ff, QS/W$		.35	.55	.10	.55	— .15	.90	.35
V, D, W, τ, VS		.42	.49	.09	.49	— .35	.91	.42
$V, D, W, \tau, VS, QS/W$		.38	.52	.10	.52	— .24	.90	.38
V, D, W	$b=0.12-0.06f$	.47	.44	.09	.44	— .50	.91	.47
$V, D, W, QS/W$		.49	.41	.10	.41	— .57	.90	.49
V, D, W, τ, ff		.38	.53	.09	.53	— .23	.91	.38
$V, D, W, \tau, ff, QS/W$		.39	.52	.09	.52	— .26	.91	.39
V, D, W, τ, VS		.47	.44	.09	.44	— .50	.91	.47
$V, D, W, \tau, VS, QS/W$		.48	.43	.09	.43	— .53	.91	.48
$V, D, (W)$	$b=0.08$	.46	.46	(.08)	.46	— .46	---	.46
$V, D, (W), \tau, ff$		.38	.54	(.08)	.54	— .22	---	.38
$V, D, (W), \tau, VS$		.46	.46	(.08)	.46	— .46	---	.46

table, but the possibility that they are part of the best group of variables has not yet been ruled out. Also, for the situation where $b=0.08$, QS/W always varies as $1-b$ or 0.92. Hence where $b=0.08$ the factor QS/W is not included in the minimizations nor in the table.

Consider first the effects of the three different constraints, for a given combination of flow variables. (Incidentally, note that in case B, it seems necessary to incorporate an empirical relation—the constraint involving b—along with the theory.) As far as predicted exponents are concerned, values of b are essentially the same for all three constraints and range from 0.08 to 0.11. The exponents m and f show some differences—mostly minor—depending on the constraint.

The average field values for the 74 stations are: for m, mean = 0.42, mode = 0.40; for f, mean = 0.50, mode = 0.48; and for b, mean = 0.08, mode = 0.07. Standard deviations are 0.13, 0.14, and 0.05 for m, f, and b, respectively. All predictions in table 5 are within a reasonable range of these average measured exponents. Thus the case B test is inconclusive. All groups of variables examined for this case, including S and QS, will therefore be considered in the next test (case C).

Case C

Case C consists of streams in which the slope at the station remains constant but the entire flow boundary is loose and readily eroded. The entire channel, in other words, is developed in noncohesive material—usually sand or sandy gravel. The loose "banks" in such channels may be reshaped from one flow to the next, and a change in discharge often alters the channel shape by erosion or deposition or both. The channel width is completely free to adjust to each new discharge.

Sixteen of the 165 stations listed in table 11 (appended to end of report) qualify for this case. Most of the 16 stations are sandy streams for which the flow data have been plotted for low-flow conditions. The entire flow boundary for these low flows would be defined as the bed during large discharges. Average exponents (arithmetic means) for the 16 stations are $m=0.21$, $f=0.26$, and $b=0.54$. Modal values are $m=0.22$, $f=0.27$, and $b=0.45$. The standard deviations are 0.07, 0.07, and 0.09 exponent units for m, f, and b respectively.

Table 6 shows the exponents predicted by the various combinations of variables, with S and QS again being omitted because they have no effect on the exponents for this case. Three of the combinations (groups 1, 4, and 6) predict b-values that are markedly discrepant from measured averages. Two groups are fairly close to the field values: (a) V, D, W, τ and ff, which predicts $m=0.22$, $f=0.30$ and $b=0.48$, and (b) V, D, W, τ and VS, which yields $m=0.25$, $f=0.25$ and $b=0.50$. Both of these combinations deserve to remain in the competition. The sixth group (V, D, W and QS/W) predicts m rather accurately (forecasting $m=0.20$) but is about one standard deviation away for both f and b. It will be included in the next test, although it was not as impressive as the two groups just mentioned for case C. Neither S nor QS nor both combined have yet been eliminated as possibly relevant.

Case B/C

The fourth major test involves stations having one firm and one loose bank (labeled B/C and C/R in table 11), with slope still constant. Fifty one of the stations are in this category.

An expression for the constraining effect of the firm bank should be incorporated into the minimum variance analysis. For the 51 applicable stations, I obtained such an expression by plotting b as a function of f. A definite trend appeared although with some scatter. Two-thirds of the plotted points fall within ± 0.11 exponent units of the value indicated by the eye-drawn best-fit line. The line through the plotted points has the equation $b=0.84-1.45f$. This relation was used in minimizing the variances of the three groups of variables remaining in the competition.

Table 6.—*Theoretical rates of change of dependent quantities with increase in discharge, for different sets of variables*

[Case C: slope constant; loose, noncoherent banks allowing complete freedom for width to adjust. Average values for 16 field sites: $m=0.21$ mean, 0.22 mode; $f=0.26$ mean, 0.27 mode; and $b=0.54$ mean, 0.45 mode]

Group No.	Dependent variables	Value of exponents						
		V Velocity (m)	D Depth (f)	W Width (b)	τ Shear (f)	ff Resistance ($f-2m$)	QS/W Power per unit bed area ($1-b$)	VS Power per unit weight (m)
1	V, D, W	0.33	0.33	0.33	0.33	—0.33	0.67	0.33
2	V, D, W, QS/W	.20	.20	.60	.20	— 20	.40	20
3	V, D, W, τ, ff	.22	.30	.48	.30	— .14	.52	.22
4	V, D, W, τ, ff QS/W	.14	.20	.66	.20	— .08	.34	.14
5	V, D, W, τ, VS	.25	.25	.50	.25	— .25	.50	.25
6	V, D, W, τ, VS, QS/W	.16	.17	.67	.17	— .15	.33	.16

Because slope is constant, the variables S and QS are not included in the minimization. Minimizing the variances of V, D, W, τ, and ff yields $m=0.28$, $f=0.27$, and $b=0.45$. For V, D, W, τ, and VS the predictions are $m=0.27$, $f=0.24$, and $b=0.49$. Minimizing the variances of V, D, W, and QS/W produces $m=0.24$, $f=0.17$, and $b=0.59$.

The average field values for the 51 stations are $m=0.30$ (mode 0.27), $f=0.31$ (mode 0.25), and $b=0.40$ (mode 0.45, poorly defined). Standard deviations are 0.10, 0.14, and 0.19 exponent units for m, f, and b, respectively.

All three groups of variables are fairly close in predicting m. For f and b the combination V, D, W, and QS/W is not as close as the other two groups. This combination also was less accurate in the previous test (case C). The combinations V, D, W, τ, and ff and V, D, W, τ, and VS again are reasonably close to the field values. The results therefore show that, for the four cases examined thus far, these latter two combinations are the only ones that consistently yield exponents close to the average observed exponents. Also, because slope has been assumed constant in the four cases examined above, there has been no way of determining whether S and (or) QS should be included in the complete set of variables. One test remains.

Case D

The fifth test includes channels in loose, readily erodible material, as with case C, but now the water-surface slope varies with discharge rather than remaining constant. Measurements for such stations are extremely scarce. Only three sets of data could be found for this case: two of them are the Wolman and Brush (1961) flume study of 0.67 mm and 2.0 mm sand, respectively, and the third is Ackers' (1964) flume study with 0.16 mm and 0.34 mm sand.

Table 7 shows the measured hydraulic exponents for the three flume studies. Ackers' values are those published in his paper and were determined by least squares. His graph of slope versus discharge showed sufficient scatter that he decided no relation could be defined; however, the plot strongly suggests a negative value for the exponent z. To get the Wolman and Brush exponents, I plotted their experimental data and, except for the z-values (determined by least squares), drew lines of best fit by eye. The plots of slope versus discharge, as with the Ackers data, show a decidedly negative exponent, but the scatter is such that the values of z, while definable, are not accurate. Values of m, f, and b, among the three studies, are reasonably consistent for experiments of this type. The average exponents (arithmetic means) for the three studies are $m=0.15$, $f=0.45$, $b=0.43$, and $z=-0.54$.

Table 7.—*Measured and predicted values of hydraulic exponents for stations having variable slope, with bed and banks readily erodible*

	m	f	b	z
Measured values				
Wolman and Brush (1961), 0.67 mm sand	0.19	0.39	0.48	−0.34
Wolman and Brush (1961), 2.0 mm sand	.11	.54	.38	−.74
Ackers (1964), 0.16 mm and 0.34 mm sand	.15	.42	.43	?
Average values of the above data	.15	.45	.43	−.54
Theoretical values (minimum variance)				
V, D, W, τ, ff	0.14	0.43	0.43	−0.29
V, D, W, τ, ff, S	.19	.35	.46	−.11
V, D, W, τ, ff, QS	.03	.62	.35	−.73
V, D, W, τ, ff, S, QS	.10	.50	.40	−.45
V, D, W, τ, VS	.33	.33	.34	−.33
V, D, W, τ, VS, S	.30	.30	.40	−.20
V, D, W, τ, VS, QS	.40	.40	.20	−.60
V, D, W, τ, VS, S, QS	.36	.36	.28	−.43

Inspection of the theoretical values of table 7 shows that none of the four groups having V, D, W, τ, and VS comes very close for the exponent m. Two of the other four combinations are not particularly close, either, for one or more exponents: the group V, D, W, τ, ff, and QS is 0.17 exponent units off for f and -0.12 units off for m, while the group V, D, W, τ, ff, and S is 0.10 exponent units low for f and somewhat high for z.

The remaining two of the eight groups come closest to the observed values. If the measured z values are reliable, then the combination V, D, W, τ, ff, S, and QS is closest ($-.05$, $+.05$, $-.03$, and $+.09$ exponent units off, for m, f, b, and z, respectively). If less weight is given to the z values which were not as well defined as the other exponents, then the group V, D, W, τ, and ff is slightly closer. For some at-a-station cases, where slope is constant, the choice is irrelevant since S and QS drop out in the minimization calculations, leaving just V, D, W, τ, and ff. Tentatively, however, the available data suggest that one group of variables, namely V, D, W, τ, ff, s, and QS, applies to all at-a-station situations.

Table 8 shows the extent to which a minimum variance analysis, with V, D, W, τ, ff, S, and QS as the appropriate group of variables, predicts the average measured exponents for the five cases.

To summarize the findings thus far: (1) the minimum-variance theory closely predicts the average hydraulic exponents for the five types of stream cross sections examined; and (2) the group of variables that consistently gives the most accurate results is V, D, W, τ, ff, S, and QS.

TABLE 8.—*Comparison of average measured exponents to exponents predicted by the minimum variance theory, using* V, D, W, τ, ff, S, *and* QS *as the appropriate variables*

[Dashed entry following case means column does not apply]

Case	Constraints	Number of applicable stations	Velocity exponent m			Depth exponent f			Width exponent b			Slope exponent z		
			Data		Theory	Data		Theory	Data		Theory	Data		Theory
			Mean	Mode		Mean	Mode		Mean	Mode		Mean	Mode	
A	Width and slope constant ($b \approx 0$).	22	0.47	0.46	0.43	0.52	0.55	0.57	---	---	---	---	---	---
B, B/R, R	Banks firm but not vertical; slope constant	74	.42	.40	---	.50	.48	---	0.08	0.07	---	---	---	---
	$b = 0.19f$	--	---	---	.36	---	---	.53	---	---	0.11	---	---	---
	$b = 0.12 - 0.06f$	--	---	---	.38	---	---	.53	---	---	.09	---	---	---
	$b = 0.08$	--	---	---	.38	---	---	.54	---	---	.08	---	---	---
C	Slope constant	16	.21	.22	.22	.26	.27	.30	.54	.45	.48	---	---	---
B/C, C/R	One bank firm; slope constant; $b = 0.84 - 1.45f$	51	.30	.27	.28	.31	.25	.27	.40	[1].45	.45	---	---	---
D	None	3 (flumes)	.15	(2)	.10	.45	(2)	.50	.43	(2)	.40	—0.54	(2)	—0.45

[1] Poorly defined.
[2] Insufficient data.

HYDRAULIC EXPONENTS OF INDIVIDUAL STATIONS

The original intent of the minimum-variance approach was to find only the group averages. However, one of the eventual goals should be an accurate prediction of the exponents for any given station.

The hydraulic exponents at any given stream cross section probably depend to some extent on certain local features. Such features might reflect the channel shape, size, slope, boundary material, and other characteristics. Information of this sort was collected for all 165 stations to see if such special data can help provide the hydraulic exponents for any stream cross section.

Three approaches for predicting exponents were explored: (a) minimum variance, (b) empirical equations based on the data of this study, and (c) the Gauckler-Manning and Chezy relations.

COLLECTION OF SPECIAL DATA

THE WIDTH-VERSUS-AREA RELATION

An approximate b-versus-f relation applicable to each individual station can be obtained by plotting estimated water-surface widths and their associated estimated cross-sectional flow areas. Cross-sectional flow area $A = DW$. Thus, where power relations exist,

$$A \propto Q^{b+f} \text{ and } W \propto A^{\frac{b}{b+f}}.$$

So, for the range over which hydraulic exponents are defined, a logarithmic graph of width-versus-flow area will show a straight line having slope $b/(b+f)$.

The value of $b/(b+f)$ permits a definition of b in terms of f, or vice versa. For instance, if $b/(b+f) = 0.20$, then $b = 0.25f$.

Another advantage to such a width-versus-area graph is that the range of width and of area (and hence also of mean depth) over which the hydraulic exponents are valid will be readily shown by the range over which the straight-line relation holds.

Points plotted to define hydraulic geometry relations represent separate and individual flow measurements. A plot of W versus A, therefore, could also be defined from separate flow measurements. However, with stable channels, the gross channel dimensions and general shape are reasonably constant with time. Thus, the relation between water-surface widths and flow areas should be approximately the same for a series of separate flows as for various hypothetical flows within any one cross section profile, as long as the selected profile is representative or typical of the general channel shape and dimensions. The validity of this assumption is tested below.

Because the channel shape and hence the widths and areas change slightly from one flow to the next, more than one cross-sectional measurement is needed to get a reliable representation of a W-versus-A relation. The experience of this study is that about three measured cross sections (taken at the same location at least several weeks apart) are needed for stations with firm, stable banks, but four to six cross sections are needed for stations where the entire flow boundary is cohesionless. Since widths and their respective areas are the only required data, surveys of the cross section serve as well as discharge measurements, as long as some flow has occurred between surveys. Surveys might even be better in that they can always extend up to and over the banks.

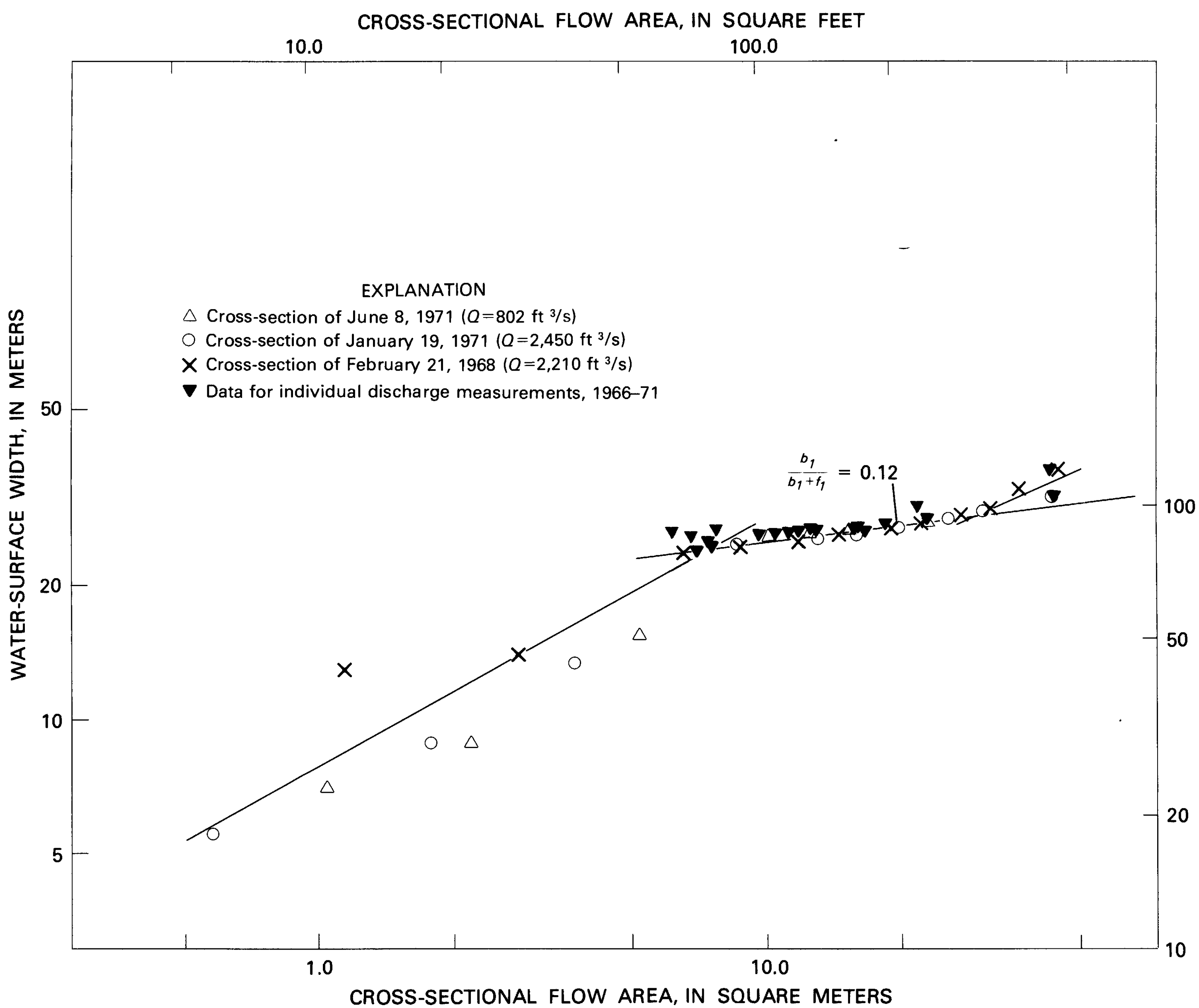

FIGURE 5.—Width versus area for a station with firm banks—White River below Tygh Valley, Oreg. Hypothetical widths and areas were generated from three cross sections which were measured at the same section on different occasions. Lines of best fit were drawn for these data. Points representing separate flow measurements added afterward for comparison.

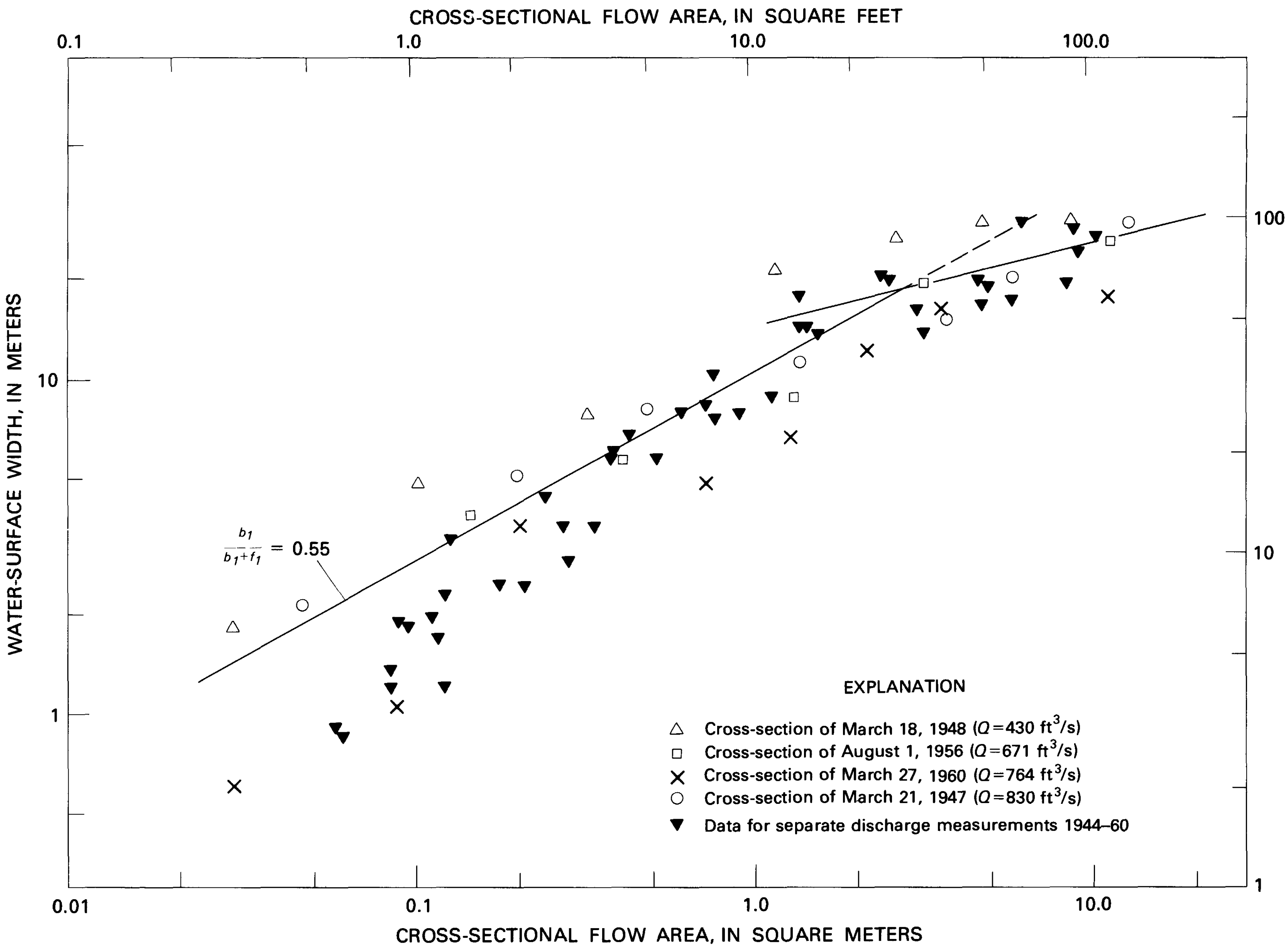

FIGURE 6.—Width versus area for a station with noncohesive boundary—Cherry Creek near Franktown, Colo. Hypothetical widths and areas were generated from four cross sections which were measured at the same site on different occasions. Lines of best fit were drawn for these data. Points representing separate flow measurements added afterward for comparison.

Figures 5 and 6 show typical W-versus-A relations for a firm-bank and loose-bank station, respectively. The standard procedure in preparing these and similar graphs for every station was as follows: (a) plot a cross section for a very high measured flow; (b) draw horizontal lines (typically 6 to 14) representing theoretical water surfaces; (c) measure with a planimeter the cross-sectional flow area associated with each water-surface width; (d) plot W versus A on log paper; (e) repeat for three or four other high-flow measurements at the same station, plotting all the data on the same graph; and (f) draw a single best-fit line by eye for the range over which a straight-line relation applies.

Two methods were tried for testing the reliability of the width-area relations. One method was to plot width against area for the many separate flows that made up the observed hydraulic-geometry relations and to compare the best-fit line with that obtained by the graphical planimeter method. This approach turned out to be infeasible because, for many stations, the range of areas for the real flows was too small. For stations having a reasonable range of measured flow areas, the widths and areas obtained by planimeter showed fair agreement with the data from real flows (figs. 5 and 6).

In the second test, the value of $b_1/(b_1+f_1)$, representing the slope of the line on the cross-section based width-versus-area graph, was compared to the "true" value of $b/(b+f)$ as computed from the observed hydraulic exponents.

Figure 7 shows the results of this test. The data are not homoscedastic, and the points are more evenly distributed on log scales, as in figure 7. The graph indicates that, as a first approximation, the value of $b/(b+f)$ can indeed be estimated by plotting widths and areas generated from cross-sectional surveys. On the other hand, the scatter undoubtedly leaves room for improvement. The biggest single discrepancy between $b_1/(b_1+f_1)$ and the "true" $b/(b+f)$ for a station is 0.35 exponent units. In a number of cases, however, the agreement is perfect. For two-thirds of the 165 cases, the estimated value $b_1/(b_1+f_1)$ is within ± 25 percent of the true $b/(b+f)$.

There is a slight tendency for the estimated $b_1/(b_1+f_1)$ to be low for stations having a high value of $b/(b+f)$, and vice versa (fig. 7).

MAXIMUM AND MINIMUM GEOMETRICAL PROPERTIES OF A SECTION

The mathematical definition of hydraulic exponents suggests that their values might be influenced by certain characteristics of the channel cross section. Take, for example, the exponent of width, b, defined as

$$b=\frac{\log W_2-\log W_1}{\log Q_2-\log Q_1}=\frac{\Delta\log w}{\Delta\log Q} \qquad (10)$$

where each subscript refers to a point on the best-fit line relating width to discharge.

A consistent way to define the two points on the best-fit line, for example, W_2 and W_1, is to take them as the maximum and minimum values, respectively, for which the straight-line relations holds. These points can be readily seen on most plots of width versus area. Figure 5 is a good example. For that station, the maximum values of width (W_{max}) and area (A_{max}) for which the power relation holds are 29.3 m (96 ft) and 29.9 m^2 (322 ft^2), respectively. Therefore, the associated depth (D_{max}) is A_{max}/W_{max} or 1.02 m (3.35 ft). Similarly, the lower end of the power relation is at a width W_{min} of 24.4 m (80 ft) and an area A_{min} of 7.52 m^2 (81 ft^2). Thus, $D_{min}=0.31$ m (1.01 ft). $\Delta\log W$ would then be $\log W_{max}-\log W_{min}=\log 96-\log 80=0.079$. $\Delta\log A$ and $\Delta\log D$ were defined in the same way for all stations.

The denominator $\Delta\log Q$ in equation 10 is not directly available. Assume the hydraulic radius is approximately equal to mean depth D and that resistance and slope are constant. A number proportional to Q_{max} (discharge at the upper end of the power relation) can then be computed from the Gauckler-Manning equation as $D_{max}^{2/3}A_{max}$. Similarly, Q_{min} would be proportional to $D_{min}^{2/3}A_{min}$. The denomination $\Delta\log Q$ then is proportional to

$$\log Q_{max}-\log Q_{min}=0.667\log\left(\frac{D_{max}}{D_{min}}\right)+\log\left(\frac{A_{max}}{A_{min}}\right).$$

Using the maximum and minimum widths, depths, and areas as just described, numbers were generated that hopefully would be proportional to the exponents b and f. The denominator in these numbers was the $\Delta\log Q$ of the previous paragraph, and the numerators were $\Delta\log W$ and $\Delta\log D$ for b and f, respectively.

For some of the loose-bank stations (such as fig. 6), no break could be discerned at the lower end of the straight-line relation on the width-area plot. For these cases, the minimum values were taken as those corresponding to a depth of 0.03 m (0.1 ft), since the current-meter method probably would not measure shallower depths with any accuracy. In a few other cases, the available cross-sectional pro-

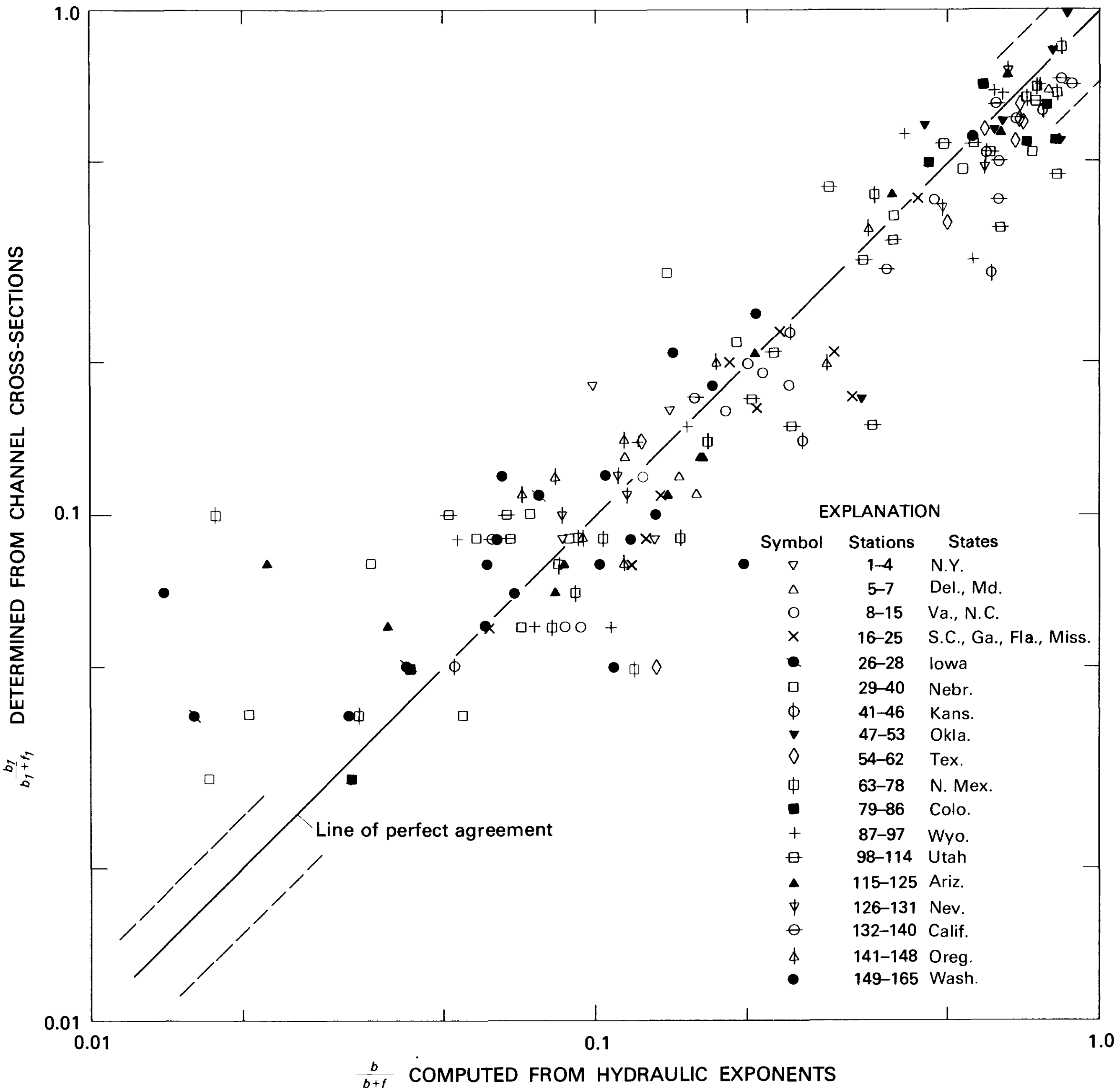

FIGURE 7.—Width-area relations based on channel cross sections compared with true values as computed from hydraulic exponents. Two-thirds of the estimated values, encompassed by the dashed lines, fall within ± 25 percent of the true values.

files did not extend high enough up the banks to show the break at maximum values; in these cases, maximum values were taken at the maximum flow conditions recorded for the site.

The maximum and minimum properties as defined here were also used to form other channel descriptors. For instance, many analysts use the channel width/depth ratio as an indicator of channel shape. In the present study, the width/depth ratio at the upper end of the applicable power relation was defined as W_{max}/D_{max}, in which these values were measured as just described. Similarly, the lower end of the power relation is associated with a width/depth ratio of W_{min}/D_{min}.

The amount of error associated with the above method of estimating the maximum and minimum

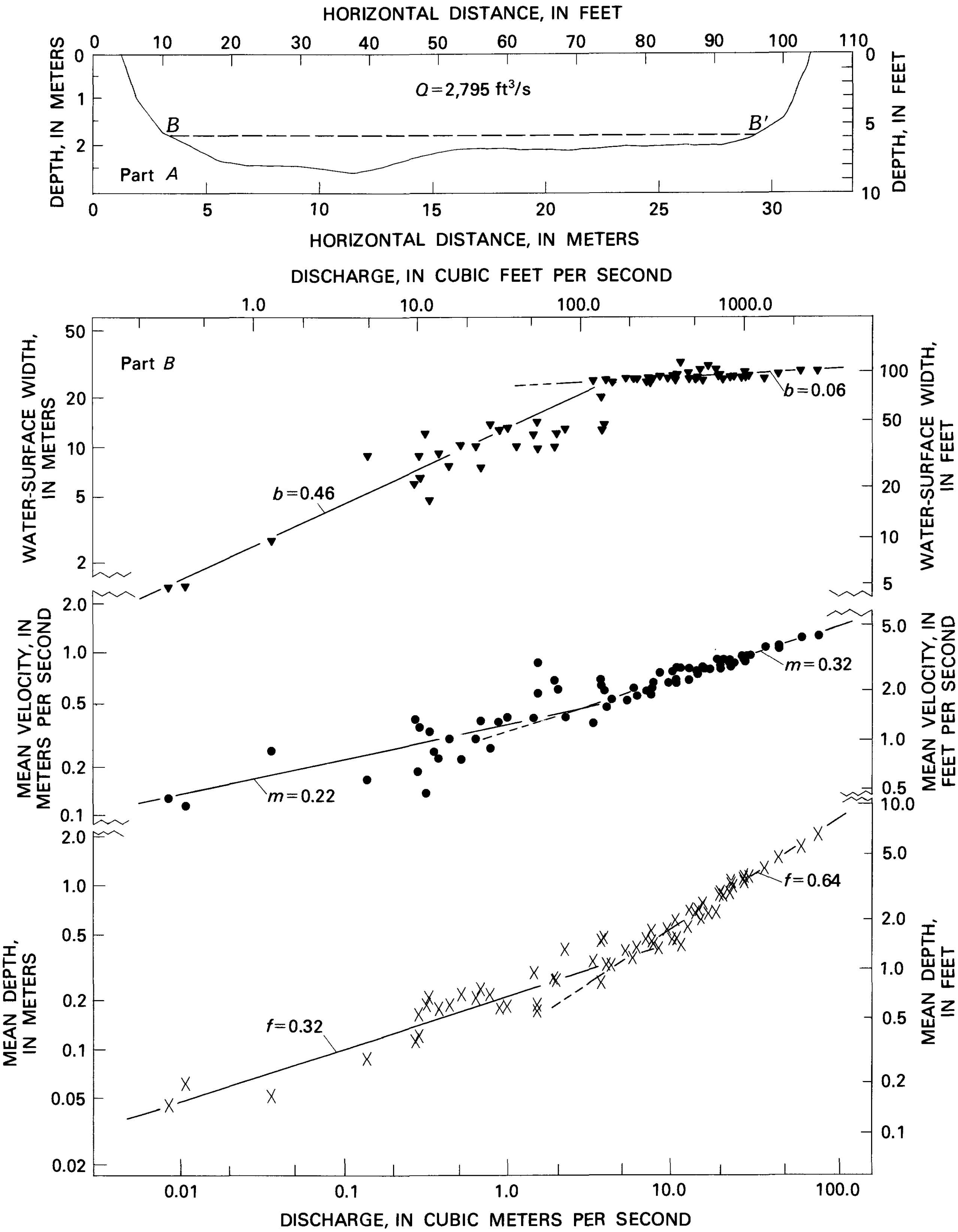

FIGURE 8.—Cross-sectional profile and hydraulic geometry, Humboldt River near Argenta, Nev.

geometrical properties of a channel section is difficult to assess. The accuracy of these values increases with the number of cross sections plotted.

Another use of maximum and minimum cross-section characteristics is to define "bank inclinations." The general steepness or flatness of the banks should affect the exponent b (Lewis, 1966; Knighton, 1974). The Humboldt River near Argenta, Nev. (station Nos. 126 and 127 of this study), shows how the inclination of the boundary, regardless of whether the latter be firm or loose, affects the hydraulic exponents. Figure 8*A* is the channel cross section at the cable, and the plotted hydraulic-geometry data are shown in figure 8*B*. Inspection of the plotted data reveals two different sets of exponents, corresponding to discharges higher and lower than about 4.25 m^3/s (150 ft^3/s), respectively. Why these different sets of exponents? From the plot of width versus discharge, the break in the exponent b is seen to occur at a water-surface width of about 26 m (85 ft). Transferring this width to the plotted profile (*B–B′* in fig. 8*A*), we see that this width just corresponds to the base of the banks, that is, to a noticeable change in the general inclination of the flow boundary. From the base of the banks up, the firm, regular boundary promotes a well-defined set of hydraulic-geometry relations, and the rather steep banks cause a relatively low value of the exponent b (0.06 in this case). At low discharges, on the other hand, the flattish and cohesionless bed is the flow boundary. Such a flattish boundary is associated with large changes in width for a given change in discharge, that is, a relatively high value of the exponent b (0.46) and more scatter on the hydraulic-geometry plots. Hence, the boundary inclination and regularity have a direct influence on the hydraulic exponents (Richards, 1976). (This example, along with figure 4 and the data in table 11, suggests that there may be some risk or questionable significance associated with computing average hydraulic exponents for a physiographic region, as some investigators do. See also Rhodes, 1977, p. 83–84.)

Maximum and minimum areas and widths, defined as explained above, were used to define a bank angle θ. The cross section was assumed trapezoidal, and the bank sections were taken as equal right triangles (fig. 9). The slope of these banks ($\bar{\theta}$) was considered to be the average bank inclination of the natural channel.

The distances used to compute $\bar{\theta}$ were the base and height of the bank sections, that is, $\tan \bar{\theta}$ = height/base. The average base of a bank section $= \frac{1}{2}(W_{max} - W_{min})$. (See fig. 9.) The height EB, considering the two bank sections equal, is

$$\frac{(A_{max} - A_{min})}{\left[\frac{(W_{max} - W_{min})}{2} + W_{min}\right]}.$$

Then for the bank inclination,

$$\tan \bar{\theta} = \frac{4(A_{max} - A_{min})}{W^2_{max} - W^2_{min}} \qquad (11)$$

Since the tangent and other trigonometric functions are sometimes awkward to use mathematically (for example, the tangent goes to infinity as the banks become vertical), $\bar{\theta}$ was expressed in degrees.

(Defining a bank angle θ by plotting the cross section and drawing a straight line by eye for the general bank inclination was too subjective, partly because the transition from bank to bed was hard to recognize on some cross sections. The above sys-

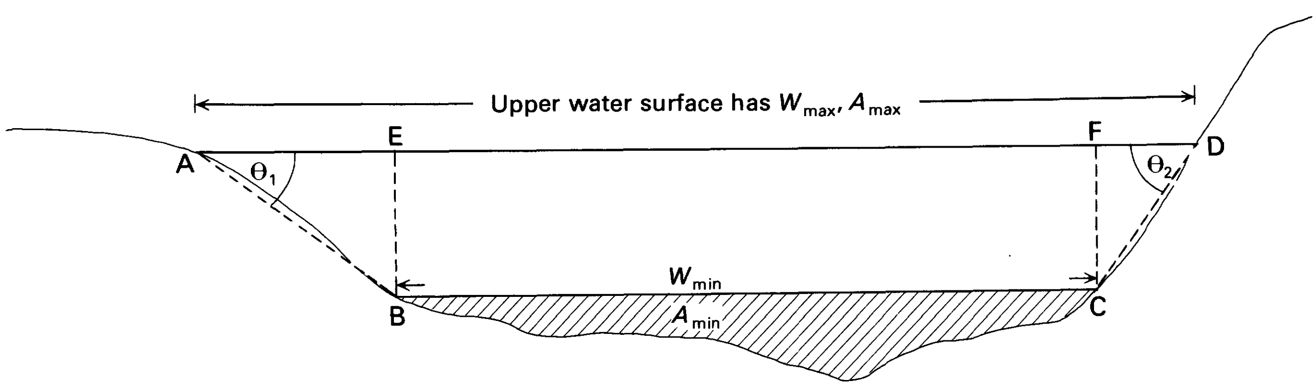

FIGURE 9.—Sketch showing concept of bank inclinations. In practice, the geometrical properties of maximum and minimum areas and widths are determined by logarithmic plotting of widths and corresponding areas measured from several cross-sectional profiles.

tem has the disadvantage that channels with various combinations of bank angles can have the same $\overline{\theta}$, so $\overline{\theta}$ is at best only an approximation. The method is consistent and objective, however.)

BED-SEDIMENT SIZES AND ESTIMATES OF BED ROUGHNESS

The bed particles for each station were sampled for size distribution at or near the cross section of interest. The median diameter, d_{50}, of the size distribution was used to describe the bed sediments. For some streams, especially with sandy beds, this value was available in published literature. At most stations, however, the particles had to be sampled. In streams on which all the bed particles were less than coarse gravel in size, representative bed samples were taken at several points across the section and combined into a single composite sample for laboratory analysis. Coarse gravel and larger particles generally were measured directly in the field by the pebble-count method (Wolman, 1954). At these sites, the finer material also was sampled and, if equal to more than about 15 percent of the total size distribution, was analyzed by sieving. The pebble count is a surface-sample frequency by number, whereas the sand-sized material was a three-dimensional sample analyzed by weight or volume frequency. On the basis of the work of Kellerhals and Bray (1971), the two distributions were combined. In making this computational union, each of the two distributions was weighted according to that percentage of stream-bed area it covered, as indicated by the pebble count.

Sieving was the usual method of size analysis for sand-sized material, though with a few samples a visual-accumulation (VA) tube was used. As the VA method is based on the principle of fall velocity rather than a direct measurement of the sieve diameter of the particle, the VA-tube data theoretically should be made comparable to sieve data by an appropriate adjustment factor (U.S. Interagency Committee, 1957, p. 37, fig. 7). However, assuming a grain-shape factor of 0.7 (naturally worn sediments), the difference in the results produced by the two methods is negligible for medium and fine sands.

Two additional size-frequency characteristics were determined for many stations: d_{84} (the grain size for which 84 percent of the distribution is finer) and a sorting measure, So, defined as $\log d_{90} - \log d_{10}$, where the subscripts indicate the percent finer in the size distribution. These data were not available for those sand-channel streams for which a median grain size was published. For certain other stations, the d_{10} was not available.

Several sources of error are associated with the median bed-particle size. One question involves where to sample, when particles of different size groups form separate patches on the streambed. Furthermore, the sizes on the bed surface may not necessarily be the same as those just beneath the surface.

The sampling procedure itself involves some error. The variability in the results for measurements made at or near the same cross section, at least with the pebble counting, can amount to about 12 percent of the median diameter (Wolman, 1954, p. 954).

In most cases, there was a time lag averaging about 1 to 2 years and ranging from about 6 months to 17 years between the period of the hydraulic measurements and the occasion of the sediment sampling. I assumed that the bed sediments did not undergo any drastic changes in size distribution during this interval.

Finally, one particle-size distribution may not apply to the full range of plotted water discharges. The size distribution of the bed particles could change with flow conditions, at least for streams with a wide range of particle sizes.

Several relative-roughness variables were studied for possible influence on the hydraulic exponents. These variables applied only to grain roughness and did not include roughness due to bed forms and channel irregularities. Examined were D_{min}/d_{50}, D_{max}/d_{50}, and the difference between these two, that is, $(D_{max}-D_{min})/d_{50}$.

Another grain-size variable investigated was $d_{50}^{1/6}$, since according to the Strickler relation, this is proportional to the Gauckler-Manning resistance coefficient in streams having coarse bed material.

Finally, according to Henderson (1966, p. 98), the quantity $(d_{50}/D)^{1/3}$ is proportional to the Darcy-Weisbach friction factor ff. Thus $(d_{50}/D_{max})^{1/3}$ would be proportional to the friction factor at maximum flow depth, $(d_{50}/D_{min})^{1/3}$ would be proportional to the friction factor at minimum depth, and the logarithmic change between these two extremes would be $\log [(D_{min}/D_{max})^{1/3}]$. These factors involve only the grain effects and do not include the influence of bed forms.

SLOPE

Channel gradients in the vicinity of the cross section were determined either from field data or topographic maps. Most field data were from longitudinal profiles of the channel or water-surface, otherwise from high-water marks on the banks or from two gages along the reach. The topographic map measurements were generally made on 7½-min quadrangle maps by measuring the horizontal distance between the contour above and the contour below the section.

The assumption that a single slope value applies to a given reach is valid only in a statistical sense. The approximate overall slope of a reach is directly related to the general topography of the area, and this average probably did not change significantly over the period of time for which the hydraulic data have been plotted. However, the actual slope, and especially the energy gradient of the flow, at any moment in time may vary with flow conditions and with channel scour and fill, at least for high discharges. Such potential variability may introduce an unknown amount of error into any relation involving slope.

There are no data available for assessing the error involved in measuring the slope with the various field methods.

Topographic map measurements introduce error both in the drawing of the map and in the measurement of the horizontal distance along the thalweg. Hack's data (1957, p. 91–93) for 64 rather steep streams in Maryland and Virginia show that slopes measured from topographic maps can differ from field-measured slopes (channel profiles measured over a distance of 61 to 152 m (200 to 500 ft)) by a factor ranging from 0.03 to nearly 15. The factors for two-thirds of his observations ranged from 0.64 to 2.00.

ACCURACY OF EXPONENTS DETERMINED BY MINIMUM VARIANCE

By assuming that the theoretical predictions of the minimum variance theory are the true mean values for each case, a rough idea of the accuracy can be obtained by looking at the spread of measured exponents about the predicted exponent. The 22 stations for case A (width and slope constant) have a standard deviation of 0.14 exponent units for m and 0.15 exponent units for f. Thus, if the distribution were normal, about 68 percent of those stations would have an exponent m that is within ± 0.14 exponent units of the predicted minimum-variance value. For the 74 firm-bank stations (case B), standard deviations are 0.14 for m, 0.14 for f, and 0.05 for b. The 16 loose-boundary stations (case C) have standard deviations of 0.07, 0.08, and 0.11 for m, f, and b, respectively. And for the 51 stations having noncohesive boundaries except for one firm bank, the standard deviations are 0.10 for m, 0.14 for f, and 0.20 for b. Thus, additional refinements to the minimum-variance analysis, probably in terms of more specific expressions of the constraints, are desirable. Another improvement would be to eliminate the subjectivity in classifying banks as "firm" or "loose."

An attempt was made to use a more objectively determined constraint, namely $b_1/(b_1+f_1)$, in minimum-variance computations (V, D, W, τ, ff) for each station. This approach obviates the need to classify banks as firm or loose, and a function of f can be substituted for b in the calculations. For control and comparison, similar computations were also made using the true b/f as given by the measured hydraulic exponents. The predicted exponents for the 165 stations have the following accuracies:

Constraint	*Exponent*	*Standard error (Exponent units)*	*Percent sums of squares explained*
true b/f	m	0.132	17
	f	.102	62
	b	.045	96
$b_1/(b_1+f_1)$	m	.131	18
	f	.117	50
	b	.082	86

(The percent of the total sums of squares of the dependent variable explained is defined as

$$100\left(1-\frac{[\text{S.E.}]^2}{\sigma^2}\right),$$

where S.E. = the standard error of the estimate of the dependent variable and σ is the standard deviation of the dependent variable.)

The above calculations suggest that using $b_1/(b_1+f_1)$ with minimum variance can provide a good estimate of b, a fair estimate of f, and a poor estimate of m.

Some reasons that might inhibit better agreement between predicted and observed exponents are:

1. A different group of variables possibly should be used in the minimization, rather than V, D, W, τ, and ff.
2. Different groups of variables possibly should be used for different hydraulic and (or) geologic situations.
3. Too many simplifying assumptions, such as constant energy gradient over the flow range, may have been adopted.
4. The minimum-variance theory may apply to certain kinds of channels and(or) flow conditions better than to others.
5. A different constraint (other than $b_1/(b_1+f_1)$) should be used.

6. The minimum variance calculations should be carried out with coefficients or weighting factors attached to each of the exponents, rather than with a single constraint applied to the entire minimization.

The first five of the above items were explored. These items deserve further study. However, no significant improvement in predicting the exponents emerged from the examination. A search for empirical coefficients (item 6) led to the surprising development that such coefficients, if known, estimate the hydraulic exponents directly, so that minimum variance apparently is not needed with this approach.

EMPIRICAL FORMULAE FOR HYDRAULIC EXPONENTS

The special data collected for this study, plus selected combinations of some variables, were tabulated for each station. The following independent variables, except when their values were unavailable, were included: S, $\overline{\theta}$, d_{50}, d_{84}, $b_1/(b_1+f_1)$, A_{max}, A_{min}, W_{max}, W_{min}, D_{max}, D_{min}, W_{max}/D_{max}, W_{min}/D_{min}, A_{max}/A_{min}, W_{max}/W_{min}, D_{max}/D_{min}, D_{max}/d_{50}, D_{min}/d_{50}, $(D_{max}-D_{min})/d_{50}$, $(W/D)_{max}/(W/D)_{min}$, $d_{50}^{1/6}$, $(d_{50}/D_{max})^{1/3}$, $(d_{50}/D_{min})^{1/3}$, $(d_{50}/D_{max})^{1/3}/(d_{50}/D_{min})^{1/3}$, the logs of all the variables just listed, So, $1/\overline{\theta}$, A_{min}/A_{max}, W_{min}/W_{max}, D_{min}/D_{max}, $(W/D)_{min}/(W/D)_{max}$,

$$\frac{f_1}{b_1+f_1}\left(=1-\left[\frac{b_1}{b_1+f_1}\right]\right),\ S^{1/2},\ \Delta\log Q$$
$$\left(=0.667\ \log\left[\frac{D_{max}}{D_{min}}\right]+\log\left[\frac{A_{max}}{A_{min}}\right]\right),$$
$\Delta\log W/\Delta\log Q$ and $\Delta\log D/\Delta\log Q$.

An equation for each hydraulic exponent was obtained by multiple regression, at a probability level of 0.05. Both the natural value and the logarithm of each exponent were tested. Each dependent variable was first regressed against the set of 59 independent variables listed in the previous paragraph. In many cases, at least one additional regression was made to reduce the equation to a more practical form of no more than two or three independent variables.

The most accurate empirical formulae are:

$$b=0.8\left(\frac{b_1}{b_1+f_1}\right) \qquad (12)$$

which has S.E.=0.082 and explains 86 percent of the sums of squares of b (fig. 10) (data are not homoscedastic, but they are not on log scales either);

$$f=0.60-0.58\left(\frac{b_1}{b_1+f_1}\right)-0.0018d_{50} \qquad (13)$$

which has S.E.=0.096 and explains 66 percent of of the sums of squares of f (fig. 11); and

$$m=0.24+0.16\ d_{50}^{1/6}-0.21\left(\frac{b_1}{b_1+f_1}\right)+0.00002\left(\frac{D_{min}}{d_{50}}\right) \qquad (14)$$

which has S.E.=0.110 and explains 45 percent of the sums of squares of m (fig. 12). In these equations d_{50} is in millimeters and D_{min} is in feet.

The coefficients in equations 13 and 14 carry appropriate units to make the equations dimensionless.

The three exponents for a cross section as given by equations 12–14 do not always add up to exactly 1.0. Equations 12–14 with the present data produce sums of exponents ranging from about 0.93 to 1.07.

These general equations for the hydraulic exponents suggest that b, the exponent of width, is almost entirely a function of the channel geometry ($b_1/(b_1+f_1)$, that is, widths and areas). The exponents f and especially m seem to depend partly on channel geometry, partly on roughness-related features, and possibly on additional features that were not studied or not measured well enough.

As with the minimum-variance test, the predictions are good for b, only fair for f and poor for m. Possible reasons why more accurate equations did not appear are:

1. The basic data, such as d_{50} and $b_1/(b_1+f_1)$, possibly were not measured accurately enough.
2. The right variables were not included in the regression. For instance, maybe sediment transport rate should be involved. Or maybe d_{50} *or* d_{84} are not the best measurements to represent grain size. Also there is a question whether the true $b/(b+f)$, rather than the measured $b_1/(b_1+f_1)$, should have been used in the regressions. (The measured $b_1/(b_1+f_1)$, of course, would have to be used in practice.) The empirical equations, especially the one for b, all become slightly more accurate if $b/(b+f)$ is used instead of $b_1/(b_1+f_1)$ in the regressions.
3. The true type of function was not found or considered.

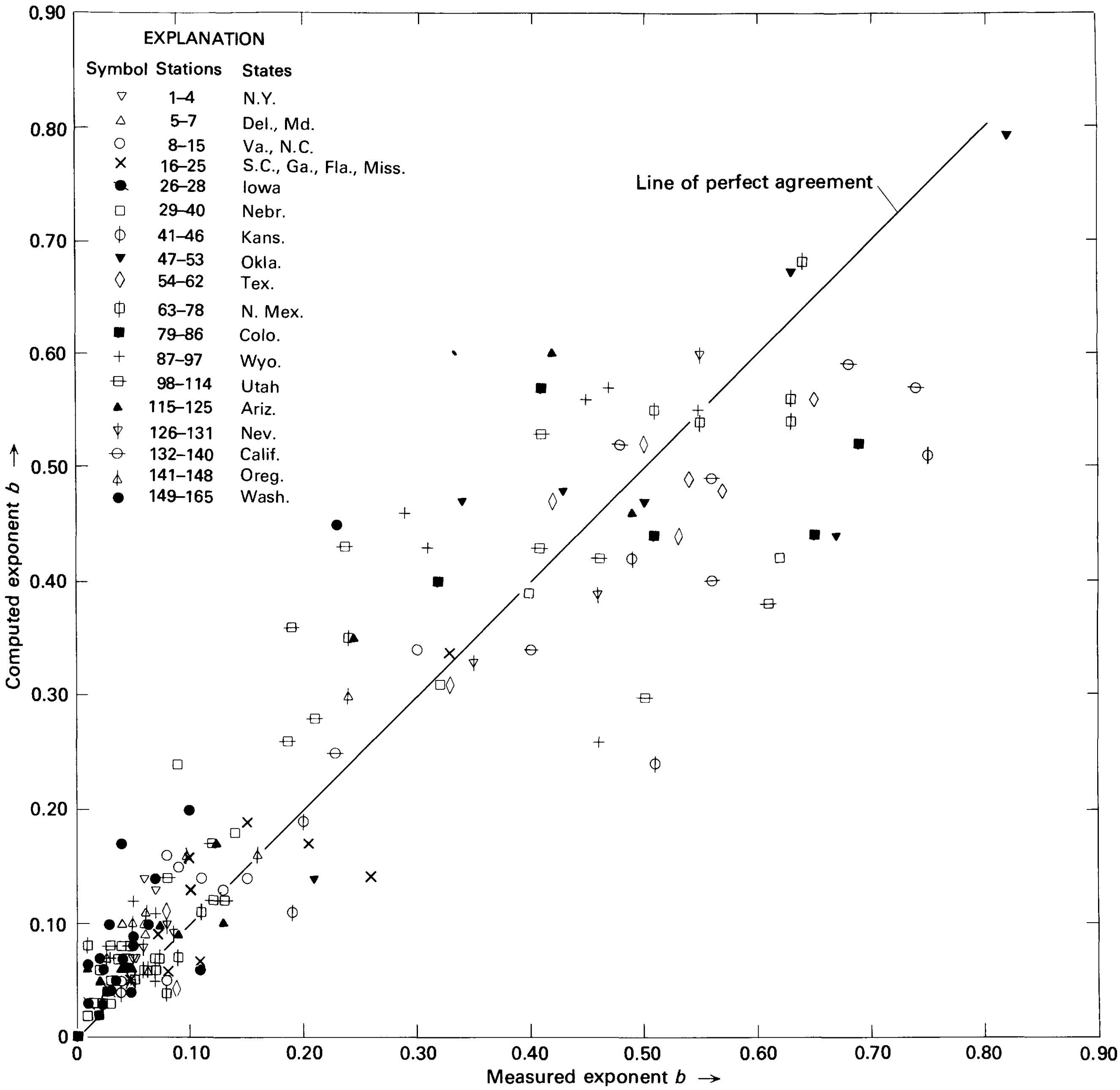

FIGURE 10.—Computed versus measured values of exponent b, where computed $b=0.8\ (b_1/(b_1+f_1))$. Standard error= 0.082 exponent units, with 86 percent of sums of squares explained.

HYDRAULIC EXPONENTS FROM THE GAUCKLER-MANNING AND CHEZY EQUATIONS

METHOD

The Gauckler-Manning and Chezy equations

$$\left(V=\frac{1.5}{n}R^{2/3}S^{1/2} \text{ and } V=CR^{1/2}S^{1/2}\right.$$

respectively, where R is hydraulic radius and n and C are roughness coefficients), can provide estimates of hydraulic exponents. Three assumptions are necessary: (a) the selected equation is valid for every station to which it is applied, (b) the roughness coefficient (Manning n or Chezy C) is constant over the flow range of interest, and (c) the slope or energy gradient is constant for the flow range of interest. The validity of at least the first two of these assumptions is doubtful for many stations on alluvial channels. Thus, the present attempt is more exploratory in nature and is motivated by a curiosity to see how close the predicted exponents come to the observed exponents.

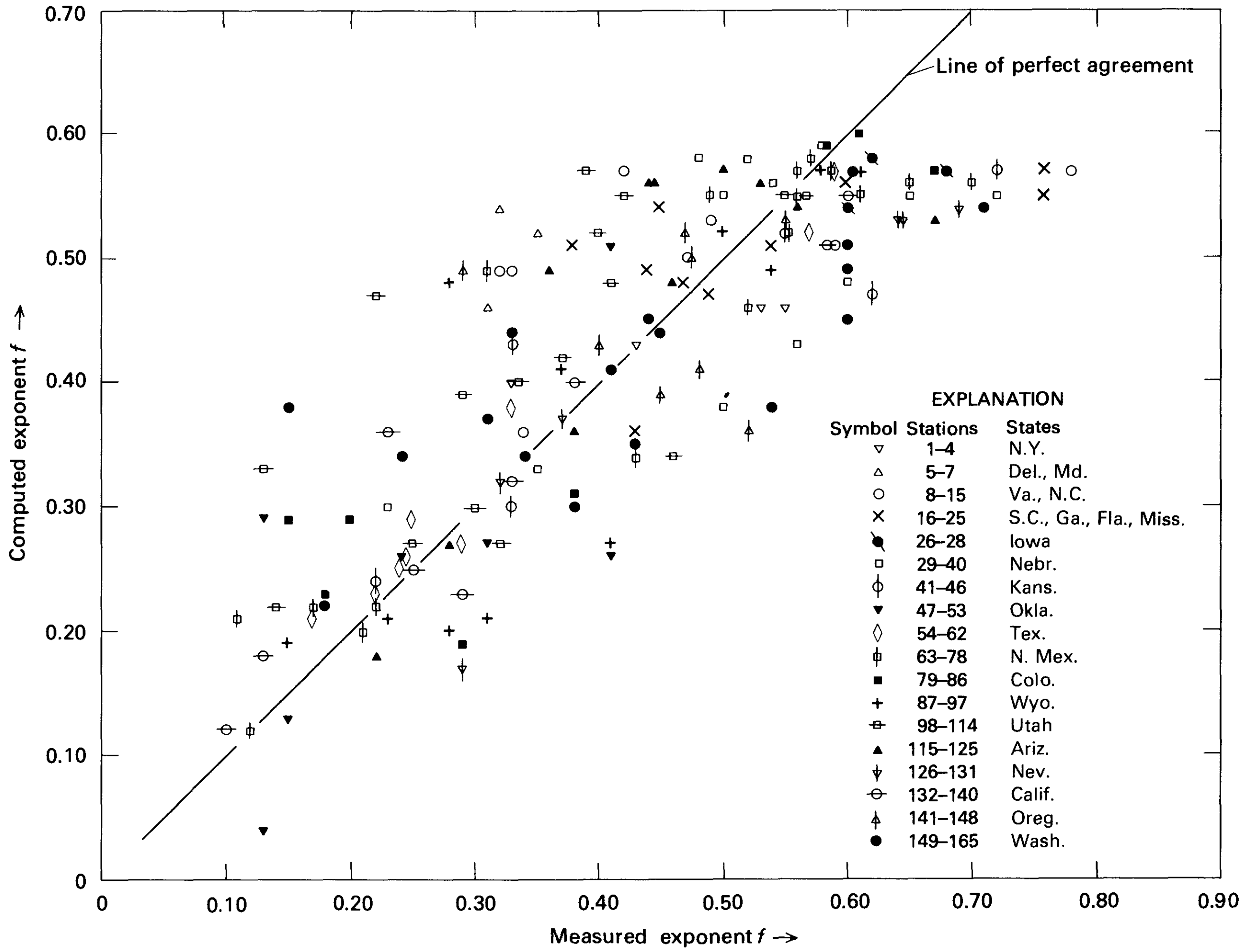

FIGURE 11.—Computed versus measured values of exponent f, where computed $f=0.60-0.58\,(b_1/(b_1+f_1))-0.0018\,d_{50}$. Standard error=0.096 exponent units, with 66 percent of sums of squares explained.

At constant roughness and slope, the Gauckler-Manning formula specifies that $V\propto R^{2/3}$. Similarly, according to the Chezy equation, $V\propto R^{1/2}$. Assuming $D\approx R$ and substituting $V=Q/A$, we get $Q\propto D^{2/3}A$ for Gauckler-Manning and $Q\propto D^{1/2}A$ for Chezy. Thus, a number proportional to Q can be computed from an associated value of D and of A.

The estimated values of W and A generated from hypothetical water-surface widths drawn on plotted channel cross sections were used as the basic data. For each width and area, the associated mean depth D was computed as A/W. The corresponding "discharge" then is $D^{2/3}A$ and $D^{1/2}A$ for the Gauckler-Manning and Chezy equations, respectively. Finally, W versus "Q" and D versus "Q" were plotted on log paper, lines of best fit were drawn by eye, and the exponents measured graphically as usual. (The depth plot probably is partly spurious, in that D is plotted against $D^{2/3}A$.) This procedure was followed for all 165 stations. Only the exponents b and f, for width and depth, respectively, were studied in this way, and m was determined as $1-b-f$.

RESULTS

For all 165 stations the standard errors and percents of total sums of squares explained, for each exponent, are as follows:

Exponent	*Formula*	*Standard error (exponent units)*	*Percent of total sums of squares explained*
m	Gauckler-Manning	0.134	15
	Chezy	.164	0
f	Gauckler-Manning	.133	35
	Chezy	.159	7
b	Gauckler-Manning	.109	75
	Chezy	.105	77

Separating the results into firm-bank and loose-bank categories did not bring about any significantly better accuracy in predicting the hydraulic exponents.

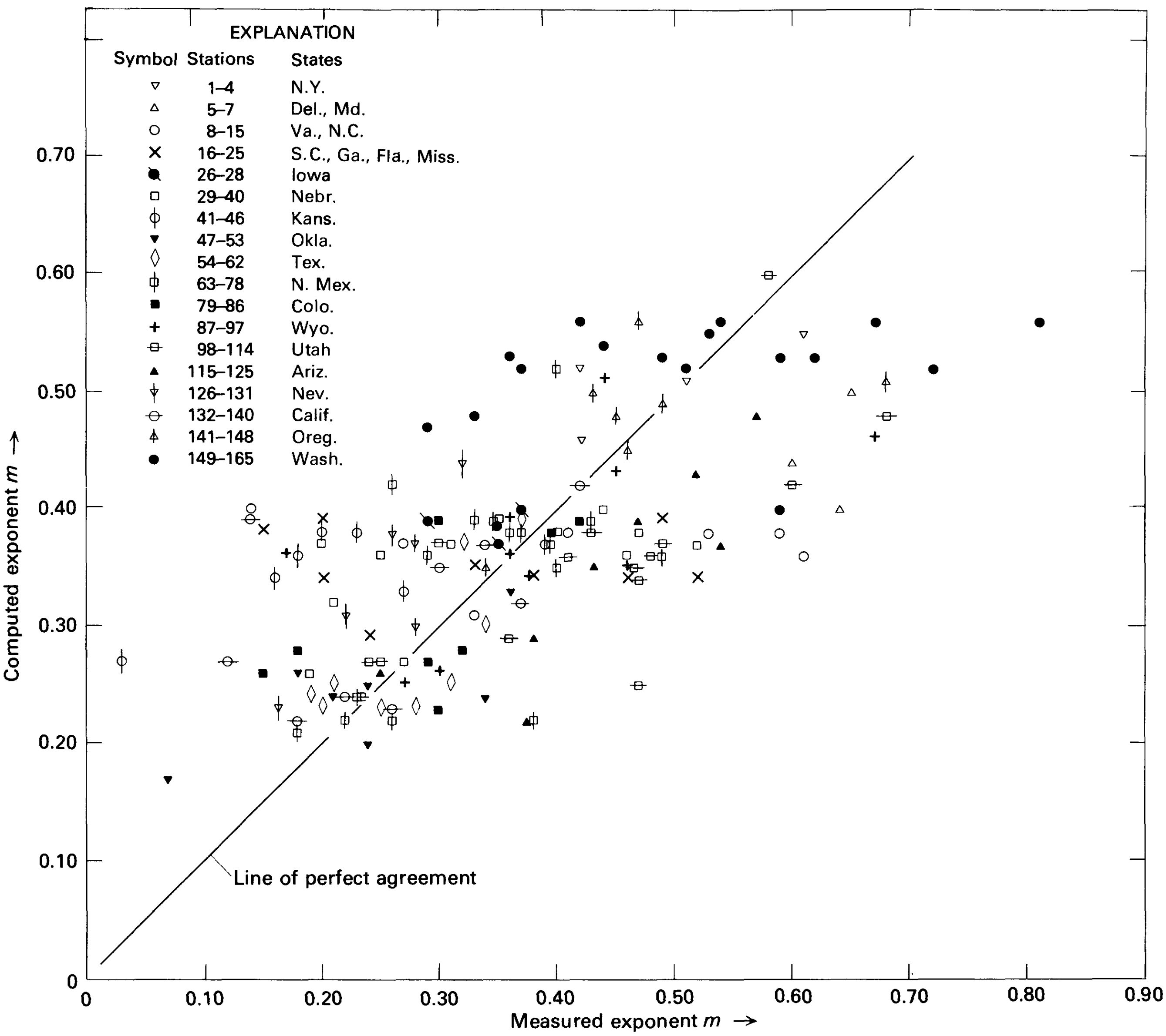

Figure 12.—Computed versus measured values of exponent m, where computed $m=0.24+0.16d_{50}^{1/6}-0.21\ (b_1/(b_1+f_1))+0.00002\ (D_{min}/d_{50})$. Standard error=0.110 exponent units, with 45 percent of sums of squares explained.

The Gauckler-Manning equation comes closer to estimating the exponents f and m than does the Chezy equation. (Standard errors for the Gauckler-Manning equation are lower, with a greater variance of the dependent variable explained.) The two flow equations are about equally accurate for the exponent b.

Both formulae, however, are somewhat less accurate in predicting hydraulic exponents than the minimum variance and empirical methods. The following section compares the results of these various methods.

COMPARISON OF METHODS OF COMPUTING HYDRAULIC EXPONENTS

Table 9 compares the statistical accuracy of the minimum variance (using $b_1/(b_1+f_1)$), empirical and Gauckler-Manning methods of predicting the hydraulic exponents for the 165 cross sections of this study. The minimum-variance and empirical methods are about equally reliable for b. The empirical equations, while still in need of improvement, are the best of the three methods for f and m. This is not surprising, since the empirical equations were derived solely from the present data. However, the

TABLE 9.—*Accuracy of methods of predicting hydraulic exponents*

Method	Exponent		
	m	*f*	*b*
Minimum Variance (using $b_1/(b_1+f_1)$):			
Standard error	0.131	0.117	0.082
Percent sums of squares explained	18	50	86
Empirical Equations:			
Standard error	.110	.096	.082
Percent sums of squares explained	45	66	86
Gauckler-Manning:			
Standard error	.123	.133	.109
Percent sums of squares explained	15	35	75

present data do cover a wide range of river conditions.

SUMMARY AND CONCLUSIONS

The minimum-variance theory assumes that in response to changes in water discharge, the adjustments in the major dependent variables tend to be as conservative as possible. These adjustments are reflected by the exponents in power relations between water discharge and each dependent variable.

Previous work done on this theory (Langbein, 1964, 1965; Scheidegger and Langbein, 1966) has not explored the question of which variables are important—a question which must be resolved if the theory is to be applied. Also, no extensive testing of the theory has previously been carried out with field data. The 165 alluvial-channel cross sections of the present study provide tentative answers to these two problems. The data have the following ranges for hydraulic exponents: $0.00 \leq b \leq 0.82$, $0.10 \leq f \leq 0.78$, and $0.03 \leq m \leq 0.81$.

Results suggest that the major dependent variables in regard to flow adjustment at channel cross sections are mean velocity, water-surface width, mean depth, shear stress, friction factor, slope, and stream power. To the extent that slope remains constant at a channel cross section, the last two of this group can be dropped from consideration. For the five types of channels studied, minimum-variance calculations with these dependent variables produce values of the exponents *m*, *f*, *b*, and *z* that are reasonably close to the average exponents found for four natural-stream cases and for one flume case. The agreement suggests some promise for the theory.

Three methods—minimum variance, empirical relations derived from the present data, and the Gauckler-Manning formula—were examined for accuracy in predicting the hydraulic exponents at any given channel cross section. This phase of the study required the collection of such special data as bed-sediment sizes, channel slope, and various geometrical properties of the channel cross section. The most accurate way to determine the at-a-station hydraulic exponents for the present data is with the empirical relations (equations 12–14). Using the equations, the predicted exponents agree with the measured exponents to the following extent: for *m*, the standard error S.E. is 0.110 exponent units with 45 percent of the sums of squares of *m* explained; for *f*, S.E.=0.096 with 66 percent of the sums of squares of *f* explained; and for *b*, S.E.=0.082 with 86 percent of the sums of squares of *b* explained. These figures show that the equations provide only a rough approximation of observed hydraulic exponents. Many reasons could easily explain the differences between predicted and observed values.

To get the important channel characteristics, the investigator needs the median diameter of the bed material and the relation between water-surface width and cross-sectional flow area for all flows up to bankfull. The latter relation can be estimated from the data of at least three cross-sectional profiles. (As many as six profiles are recommended for stations in loose, sandy materials.) All profiles should be measured at the same section at time intervals of several weeks or months.

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STATISTICAL VARIANCE AND A HYDRAULIC EXPONENT
AND
SUMMARY OF DATA

STATISTICAL VARIANCE AND A HYDRAULIC EXPONENT

This section explains the close relation between the square of a hydraulic exponent and the term "variance" of conventional statistics.

Consider a series of discharge measurements and the associated mean velocities, all taken at the same cross section on a stream. Change these measured values into logarithms and let σ = the standard deviation of the logarithms of each hydraulic property. For example, $\sigma_{\log V}$ is the standard deviation of the log-velocity values, a measure of the spread of the log-velocity values about the mean of the log-velocity group. The following hypothetical example will show that the hydraulic exponent of velocity, m, can be defined as

$$m=r\left(\frac{\sigma_{\log V}}{\sigma_{\log Q}}\right)$$

where r is a correlation coefficient (Crow and others, 1960, p. 158).

Suppose we have 10 different discharge measurements and a mean velocity for each discharge. It is specified that these two variables have a power relation, such as $V \propto Q^m$ or $\log V \propto m\ (\log Q)$. Figure 13 is a graph of the data for this hypothetical example. The exponent m (the slope of the line) can be measured graphically and equals 0.5 for these data. The ratio of the standard deviations, $\sigma_{\log V}/\sigma_{\log Q}$, should also equal 0.5. Columns 2 and 5 of table 10 give the measured values of log V and log Q, respectively. The standard deviations of the log V values ($\sigma_{\log V}$) and of the log Q values ($\sigma_{\log Q}$) are calculated in table 10 according to the usual procedure (Crow and others, 1960, p. 12). The values for this example come out to be $\sigma_{\log V}=0.496$ and $\sigma_{\log Q}=0.992$. Then the ratio

$$\frac{\sigma_{\log V}}{\sigma_{\log Q}}=\frac{0.496}{0.992}=0.5=m.$$

For this example, all the points lie on a straight line, so the correlation coefficient is 1.0 The correlation coefficients for velocity, depth, and width are rarely 1.0 in regression analyses but are usually higher than about 0.7. For example, the following correlation coefficients have been computed from Culbertson and Dawdy's (1964) data:

	Rio Grande at San Felipe, N. Mex.	*Rio Grande near Bernalillo, N. Mex.*	*Rio Grande at Cochiti, N. Mex.*
Velocity	1.00	0.99	0.98
Depth	.99	.99	.98
Width	(1)	(1)	.82

[1] Constant.

In some cases, it is therefore acceptable to assume that the correlation coefficients of the various hydraulic exponents are approximately equal or constant. Thus, just as $m \propto \sigma_{\log V}/\sigma_{\log Q}$, as shown by this example, the same principle applies to each of the other dependent variables, so that

$$f \propto \frac{\sigma_{\log D}}{\sigma_{\log Q}},\ b \propto \frac{\sigma_{\log W}}{\sigma_{\log Q}},\ \text{and so forth.}$$

All of the latter relations contain $\sigma_{\log Q}$ as a common factor. The proportionalities, therefore, are still valid if $\sigma_{\log Q}$ is deleted. This leaves $f \propto \sigma_{\log D}$, $b \propto \sigma_{\log W}$, and so forth. In other words, each hydraulic exponent is proportional to the standard deviation of the logarithms of its respective hydraulic property. Now square both sides of these proportionalities: $f^2 \propto \sigma^2_{\log D}$, $b^2 \propto \sigma^2_{\log W}$, and so forth. Since "variance" is the square of a standard deviation, the square of a hydraulic exponent is proportional to the variance of the logarithms of the associated hydraulic property. This is why "variance" is used as a shorthand term for "square of hydraulic exponent."

The above statistical definitions of the hydraulic exponents are not always valid. Some of the questionable procedures concerning the logarithms of the several hydraulic properties are the following:

1. They are treated as if they are distributed at random, that is, not predictable for any given case. Actually the hydraulic properties, rather than being randomly distributed, follow definite laws and so should be predictable on the basis of these laws. The values are not predictable at present due to insufficient knowledge. However, as explained on pages C1 and C2 of the Scheidegger and Langbein (1966) paper, the net result of many predictable actions often is the same as if the whole process were random. The process, therefore, lends itself to a statistical approach.
2. They are assumed to be mutually independent. The reason for this assumption is unclear and may depend on the definition of independence. Certainly the hydraulic properties (V, D, and W, for example) are related in that a change in one property usually is associated with changes in the other factors.
3. They are assumed to have approximately normal distributions. A computed standard deviation is most meaningful only if the distribution is approximately normal. There is no basis for assuming that the values are either normally or

TABLE 10.—*Standard deviations of log* V *and of log* Q *(see fig. 13)*

Point on graph	Log *V*	Deviation from mean	(Deviation)^2	Log *Q*	Deviation from mean	(Deviation)^2
1	−0.45	−0.74	0.5476	0.10	−1.48	2.1904
2	−.30	−.59	.3481	.40	−1.18	1.3924
3	−.15	−.44	.1936	.70	−.88	.7744
4	+.05	−.24	.0576	1.10	−.48	.2304
5	.25	−.04	.0016	1.50	−.08	.0064
6	.40	+.11	.0121	1.80	+.22	.0484
7	.55	.26	.0676	2.10	.52	.2704
8	.70	.41	.1681	2.40	.82	.6724
9	.85	.56	.3136	2.70	1.12	1.2544
10	1.00	.71	.5041	3.00	1.42	2.0164
Sums	2.90	---	2.2140	15.80	---	8.8560
Means	.29	---	-----	1.58	---	-----

$$\sigma_{\log V}=\sqrt{\frac{\Sigma(\text{deviations})^2}{N-1}}=\sqrt{\frac{2.2140}{9}}=0.496$$

$$\sigma_{\log Q}=\sqrt{\frac{\Sigma(\text{deviations})^2}{N-1}}=\sqrt{\frac{8.8560}{9}}=0.992$$

symmetrically distributed. Measured data could resolve this question.

One or more of the above three conditions probably is not satisfied in the field. The importance of this in regard to the statistical derivation of the exponents is uncertain. The statistical derivation relating the exponents to the variances of the respective hydraulic properties would, therefore, benefit from further study and clarification. However, such a derivation is not vital to the general minimum-variance theory. The theory merely proposes that the most probable exponents are those whose squares add up to the smallest number, because this situation corresponds to the most uniform distribution of the imposed change, as shown earlier. The association with the variance of conventional statistics is only of minor importance.

(For table 11 see p. 42.)

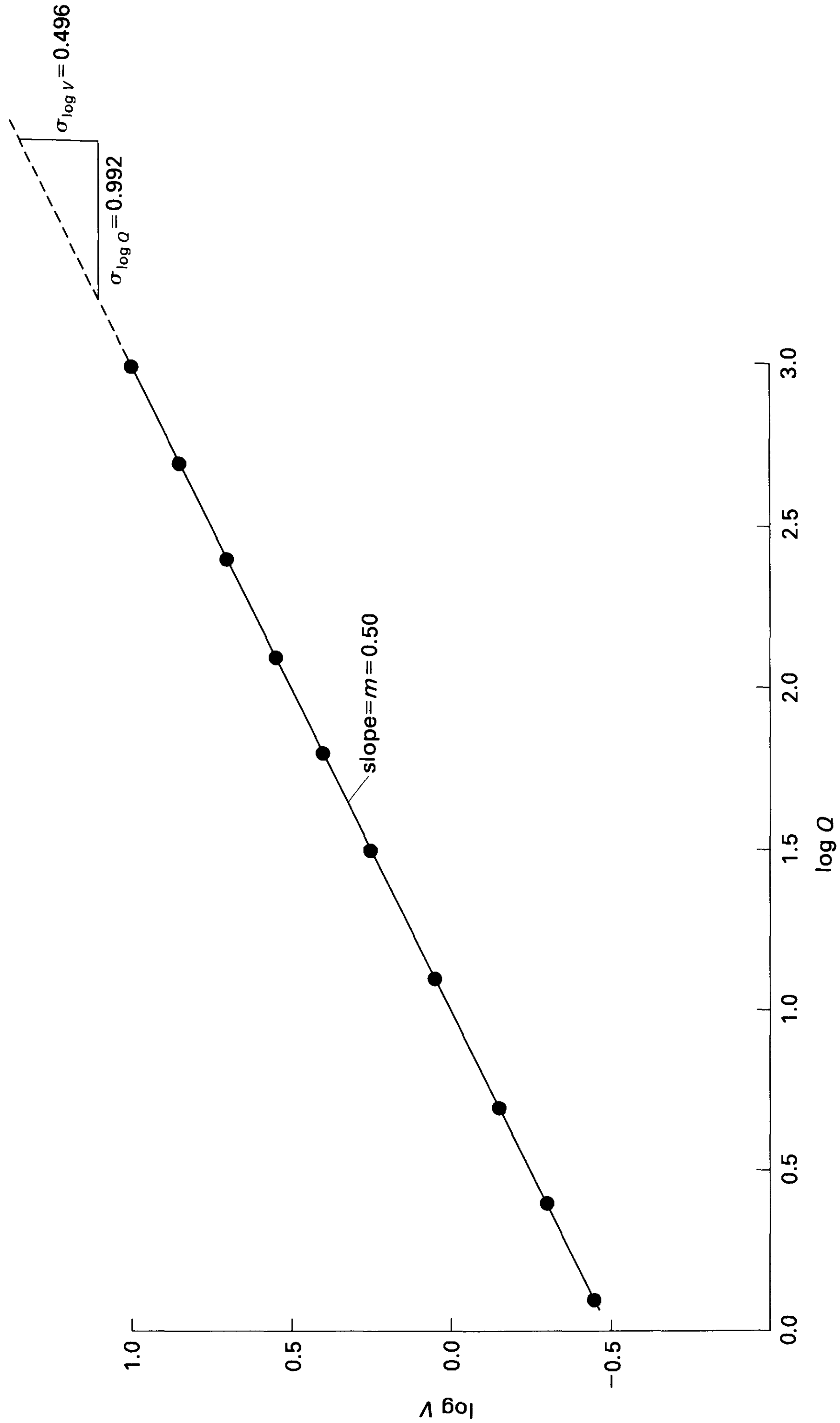

FIGURE 13.—Power relationship between velocity and discharge, where the exponent=0.5.

TABLE 11.—

Station No.	Station	Hydraulic exponents m	f	b	Minimum variance case [1]	Log cycles of Q on plot	Amount of scatter on hydraulic geometry plots: Velocity (percent)	Depth (percent)	Width (percent)
1	Chenango River near Chenango Forks, N.Y	0.51	0.43	0.07	B	1.0	2/2	5/4	2/4
2	Genesee River near Mt. Morris, N.Y	.42	.55	.06	B	1.4	12/10	10/10	18/12
3	Genesee River at Rochester, N.Y	.61	.33	.05	B	1.3	25/25	24/24	8/6
4	Susquehanna River at Colliersville, N.Y	.42	.53	.05	B	1.0	12/5	12/12	7/11
5	White Clay Creek near Newark, Del	.65	.31	.04	B	.8	12/8	14/10	3/4
6	Big Elk Creek at Elk Mills, Md	.64	.32	.06	B	1.3	42/24	41/32	6/8
7	Murderkill River near Felton, Del	.60	.35	.06	B	1.2	36/14	23/12	7/17
8	Rappahannock River at Remington, Va	.39	.47	.15	B	1.2	30/18	15/14	7/11
9	Neuse River at Kinston, N.C	.61	.32	.08	B	1.2	20/16	17/16	7/7
10	Middle Creek near Clayton, N.C	.33	.34	.30	B	2.1	18/29	27/28	22/16
11	Nahunta Swamp near Shine, N.C	.53	.42	.04	B	1.2	47/25	42/30	12/12
12	Contentnea Creek at Hookerton, N.C	.27	.59	.13	B	1.0	9/16	16/10	7/6
13	Fishing Creek near Enfield, N.C	.41	.49	.07	B	.8	7/9	8/6	7/3
14	Trent River near Trenton, N.C	.59	.33	.09	B	1.1	5/4	9/6	6/3
15	Hyco River at McGehees Mill, N.C	.14	.78	.08	B	1.0	19/19	25/12	7/4
16	Savannah River at Augusta, Ga. (Butler Cr.)	.35	.60	.08	B	.7	15/10	10/13	3/3
17	Little River near Mt. Carmel, S.C	.20	.76	.05	B	1.2	24/9	10/23	6/3
18	Little River near Adel, Ga	.33	.47	.20	B	1.9	74/32	34/44	21/19
19	Rocky Creek near Dudley, Ga	.46	.44	.10	B	1.5	50/34	45/34	16/10
20	Suwannee River at Fargo, Ga	.20	.54	.26	B	1.3	26/22	25/20	29/23
21	Altamaha River at Doctortown, Ga	.49	.45	.07	B	.6	16/8	17/11	3/3
22	Alapaha River at Statenville, Ga	.15	.76	.11	B	1.0	14/10	8/10	7/6
23	Spruce Creek near Samsula, Fl	.52	.38	.10	B	1.1	29/24	40/28	13/23
24	Econfina River near Perry, Fl	.38	.49	.15	B	1.8	42/30	30/16	44/18
25	Bayou Pierre near Willows, Miss	.24	.43	.35	B	1.3	27/20	27/15	16/13
26	Nishnabotna River above Hamburg, Iowa	.35	.60	.05	B/C	2.0	14/14	15/12	5/8
27	Wapsipinicon River near DeWitt, Iowa	.29	.68	.03	A	1.0	10/18	23/7	4/2
28	Des Moines River near Tracy, Iowa	.37	.62	.01	A	1.4	11/6	8/9	2/2
29	Elkhorn River at Ewing, Nebr	.27	.35	.40	B/C	1.2	15/14	29/28	51/22
30	Elkhorn River near Waterloo, Nebr. (upper cable).	.44	.50	.04	B	1.2	20/18	21/17	4/6
31	Elkhorn River near Waterloo, Nebr. (lower cable).	.52	.48	.01	A	1.2	12/18	20/12	3/1
32	Elkhorn River at Neligh, Nebr	.31	.65	.04	B	1.0	15/15	26/12	8/11
33	Elkhorn River near Norfolk, Nebr	.35	.56	.09	B	1.9	25/13	18/21	12/18
34	Little Blue River near Deweese, Nebr	.21	.50	.32	B/C	1.2	28/26	41/18	20/14
35	West Fork Big Blue River near Dorchester, Nebr.	.25	.60	.14	B	1.5	34/40	54/26	14/8
36	Niobrara River near Cody, Nebr	.46	.54	.02	A	.8	18/16	25/13	4/3
37	Republican River at Stratton, Nebr	.19	.23	.62	B/C	3.3	28/17	48/32	60/40
38	Republican River below Harlan Co. Dam, Nebr	.40	.58	.01	A	1.0	5/10	16/6	4/3
39	Republican River near Guide Rock, Nebr	.20	.72	.07	B	1.1	14/7	13/16	19/10
40	Plum Creek at Meadville, Nebr	.47	.52	.03	A	.7	15/22	35/15	9/6
41	Smoky Hill River at Elkader, Kansas	.03	.22	.75	C	1.9	49/75	39/17	70/51
42	Beaver Creek at Cedar Bluffs, Kansas	.27	.55	.19	B/C	1.6	10/9	31/8	10/10
43	Arkansas River at Syracuse, Kansas—300 ft above gage.	.23	.72	.04	B	.6	13/9	15/14	3/17
44	Arkansas River at Syracuse, Kansas—400 ft above gage.	.20	.33	.49	B/C	1.7	20/10	48/15	24/37
45	Sappa Creek near Oberlin, Kansas—low flows	.16	.33	.51	C	2.3	50/48	47/36	65/24
46	Sappa Creek near Oberlin, Kansas—medium flows.	.18	.62	.20	B/C	1.1	13/15	16/13	10/10
47	Washita River near Durwood, Okla	.36	.41	.21	B/C	1.0	13/12	20/14	9/7
48	Wolf Creek near Fargo, Okla	.07	.13	.82	B/C	1.2	17/20	31/21	47/35
49	Canadian River near Noble, Okla	.34	.24	.43	C	2.9	20/25	53/20	31/28
50	Canadian River at Bridgeport, Okla	.24	.15	.63	B/C	1.9	33/22	60/34	55/46
51	North Canadian River near El Reno, Okla	.21	.31	.50	C	2.1	70/25	60/39	46/35
52	Cimarron River near Buffalo, Okla	.18	.13	.67	C	2.5	45/26	28/26	45/37
53	Cimarron River near Guthrie, Okla	.24	.41	.34	B/C	2.3	31/27	47/15	41/30
54	Wichita River at Wichita Falls, Tex	.32	.59	.09	B	.9	21/20	21/15	6/12
55	Wolf Creek at Lipscomb, Tex	.31	.29	.42	C	2.3	40/20	46/22	76/50
56	Canadian River at Tascosa, Tex	.28	.22	.50	B/C	1.9	27/31	40/34	65/36
57	Canadian River near Amarillo, Tex	.20	.17	.65	B/C	1.9	23/28	64/32	72/43
58	Red River near Quanah, Tex	.21	.25	.53	B/C	2.6	52/30	41/30	60/46
59	Prairie Dog Town Fork Red River near Lakeview, Tex.	.19	.24	.57	C	4.1	53/47	95/28	80/46
60	Prairie Dog Town Fork Red River near Childress, Tex.	.25	.24	.54	C	2.5	87/50	78/34	110/53
61	Brazos River near South Bend, Tex	.37	.57	.08	B/C	1.4	32/18	30/27	9/22
62	Brazos River at Seymour, Tex	.34	.33	.33	B/C	2.1	47/36	45/35	75/42
63	Rio Chama near Chamita, N. Mex	.36	.43	.24	B/C	1.4	41/25	31/26	31/22
64	Rio Grande Floodway at San Marcial, N. Mex. —cable.	.40	.56	.05	B	.9	8/13	21/4	3/4
65	Rio Grande Floodway at San Marcial, N. Mex. —200–300 ft above gage.	.18	.21	.63	B/C	2.6	42/27	34/24	30/41
66	Rio Grande at Otowi Bridge near San Ildefonso, N. Mex.	.37	.55	.11	B	.9	25/19	27/20	10/19
67	Canadian River at Logan, N. Mex	.39	.57	.02	A	.8	5/7	8/4	3/2
68	Revuelto Creek near Logan, N. Mex	.26	.70	.07	B	1.1	22/13	21/14	8/5
69	Pecos River near Acme, N. Mex.—low-intermediate flows.	.38	.11	.51	B/C	2.0	38/22	60/48	70/32
70	Pecos River near Acme, N. Mex.—cable	.43	.56	.01	A	.8	11/10	13/11	11/4
71	Pecos River near Artesia, N. Mex	.29	.65	.06	B	1.2	55/18	30/36	12/14
72	San Juan River at Farmington, N. Mex	.35	.61	.07	B	1.1	50/21	25/27	10/5
73	San Juan River near Shiprock, N. Mex	.49	.49	.05	B/R	1.1	27/20	24/19	9/5
74	Gila River near Gila, N. Mex	.40	.52	.09	R	.8	3/4	7/3	3/4
75	Gila River near Redrock, N. Mex.—low flows	.26	.12	.64	C/R	1.3	40/20	30/20	33/24
76	Gila River near Redrock, N. Mex.—high flows	.33	.59	.08	R	.8	4/9	15/20	13/16
77	Rio Salado near San Acacia, N. Mex	.22	.17	.63	C	3.4	31/27	40/18	46/30
78	San Francisco River near Alma, N. Mex	.23	.22	.55	C	3.1	15/36	60/32	38/36
79	Cherry Creek near Melvin, Colo. (1940–60 site)	.29	.20	.51	B/C	2.9	32/40	56/41	100/42
80	Cherry Creek near Melvin, Colo. (1960–68 site)	.32	.38	.32	B/C	2.8	30/18	47/27	56/42
81	Cherry Creek near Franktown, Colo	.18	.15	.65	C	2.0	42/25	47/28	50/35
82	Kiowa Creek at Kiowa, Colo	.42	.58	.02	A	1.9	20/12	20/13	10/16

See footnotes at end of table.

Summary of data

Slope S (ft/ft)	Average bank angle $\bar{\theta}$ (degrees)	d_{50} (mm)	d_{84} (mm)	Sorting So	$\frac{b_1}{b_1+f_1}$	A_{max} (ft²)	A_{min} (ft²)	W_{max} (ft)	W_{min} (ft)	D_{max} (ft)	D_{min} (ft)
0.0008	6.0	47	77	0.875	0.16	4000	94	404	222	9.9	0.4
.0005	19.4	21	----	----	.18	1800	202	168	114	10.7	1.8
.0002	23.6	85	120	.567	.09	2870	225	242	192	11.8	1.2
.0020	22.1	50	93	.967	.09	830	275	160	143	6.1	1.9
.0008	24.9	35	140	1.360	.13	200	51	64	54	3.1	.9
.0032	22.1	2.7	14.5	1.553	.11	126	15.8	51	41	2.4	.4
.0019	23.3	7.6	30	1.427	.12	52	6.8	30	24	1.6	.3
.0018	13.4	1.4	5.5	1.301	.18	1730	94	182	107	9.5	.9
.0001	20.2	.50	.90	.610	.20	2950	202	191	113	15.4	1.8
.0049	2.6	.83	3.3	1.326	.42	43	.93	47	9.3	.9	.1
.0009	55.6	.60	18	1.117	.06	81	24	39	37	2.0	.6
.0002	19.3	.58	1.2	.686	.16	810	162	127	98	6.3	1.7
.0008	30.3	1.1	2.8	1.233	.12	1470	63	123	84	11.9	.8
----	34.5	1.6	10	1.865	.19	760	66	72	44	10.5	1.5
.0006	53.3	1.8	4.3	1.137	.06	525	29.7	67	56	7.8	.5
----	26.3	.80	3.5	1.599	.08	9100	680	420	339	21.6	2.0
----	52.1	.80	2.1	1.152	.06	480	57	73	64	6.5	.9
.0005	18.4	.75	----	----	.21	440	52	80	51	5.5	1.0
----	11.4	.35	.53	.609	.20	101	8.5	48	29	2.1	.3
.0001	8.5	.22	.50	.699	.17	240	25.8	95	65	2.5	.4
.0003	14.2	.60	.90	.431	.11	4300	820	415	349	10.3	2.3
.0004	36.8	.64	.95	.447	.09	1130	123	122	100	9.2	1.2
.0002	41.2	.17	.26	.624	.16	49	5.6	18	13.4	2.6	.4
.0004	44.9	.31	.49	.699	.23	700	26.0	50	23	14.0	1.1
.0008	1.0	.38	.55	.464	.42	113	2.5	122	25	.9	.1
.0002	32.6	.29	.40	.322	.11	2750	136	170	122	16.1	1.1
.0004	30.6	.64	1.2	.791	.05	1670	457	245	228	6.8	2.0
.0003	30.2	.53	1.0	.909	.04	6200	925	475	441	13.0	2.1
.0007	1.8	.30	.50	.602	.48	95	.8	80	8.0	1.1	.1
.0004	17.0	.24	.37	.693	.10	2050	580	282	248	7.2	2.3
.0004	22.0	.24	.50	.590	.04	1970	173	296	266	6.6	.7
.0010	28.8	.26	.45	.565	.09	162	53.0	66	60	2.4	.9
.0007	9.4	.25	.43	.572	.30	3210	605	281	170	11.4	3.6
.0014	2.0	1.1	----	----	.39	92	1.68	80	17	1.1	.1
.0002	24.2	.48	1.0	.667	.22	725	118	90	60	8.0	2.0
.0015	28.2	.27	.80	.486	.08	240	41.5	74	64	3.2	.6
.0017	1.3	.32	----	----	.52	41.5	.84	63	8.4	.6	.1
.0007	30.6	.56	.40	2.398	.03	400	24.3	127	117	3.1	.2
.0008	17.1	.44	1.0	1.029	.09	790	124	173	148	4.5	.8
.0019	53.0	.75	12	2.000	.04	248	17.0	58	52	4.2	.3
.0255	1.2	1.2	----	----	.63	8.4	.53	30	5.3	.2	.1
.0009	27.8	.20	----	----	.14	34.0	1.14	17	10.8	1.9	.1
.0014	12.5	.58	----	----	.05	415	132	212	200	1.9	.7
.0014	.9	.58	----	----	.52	104	1.1	116	11.0	.8	.1
.0013	7.7	1.4	----	----	.30	18.0	.38	19	6.2	.9	.06
.0013	28.1	1.4	----	----	.23	133	20.5	35	23	3.8	.9
.0002	10.5	.17	.23	.431	.17	1320	31.0	168	86	7.8	.4
.0009	.8	.36	.53	.450	.98	23.5	3.9	58	10.0	.4	.4
.0005	.6	.19	.27	.364	.60	3000	5.8	760	18	3.9	.3
.0018	.2	.14	.25	.590	.83	470	8.0	520	17	.9	.5
.0006	.8	.21	.31	.418	.58	24.7	.94	63	9.4	.4	.1
.0010	.3	.36	.53	.444	.55	340	2.2	335	22	1.0	.1
.0004	.5	.35	.63	.637	.59	2200	7.4	660	23	3.3	.3
.0002	40.3	.38	.60	.573	.05	825	69	127	113	6.5	.6
.0008	.5	.38	.59	.496	.58	3100	.50	780	5.0	4.0	.1
.0012	.1	.20	.28	.532	.65	3700	3.4	1530	17.5	2.4	.2
.0010	.5	.22	.31	.568	.69	215	8.0	213	22.5	1.0	.4
.0007	.4	.18	.25	.615	.55	14500	.94	1900	9.4	7.6	.1
.0020	.2	.20	.34	.699	.60	7000	4.8	1690	21.0	4.1	.2
.0011	.1	.16	.30	.869	.61	4000	.66	1780	8.5	2.2	.08
.0003	15.4	.31	.38	.312	.14	4100	530	315	233	13.0	2.3
.0006	1.2	.31	.45	.377	.38	360	26	220	81	1.6	.3
.0012	.8	10.3	31	2.714	.43	125	9.6	151	51	.8	.2
.0004	13.2	.14	----	----	.06	500	40	170	147	2.9	.3
.0004	.4	.14	----	----	.70	90	.50	147	3.7	.6	.1
.0021	36.1	.53	----	----	.14	2650	238	150	106	17.7	2.2
.0006	18.6	.18	----	----	.04	1040	157	262	242	4.0	.6
.0017	37.9	.22	.40	.626	.07	2600	495	216	191	12.0	2.6
.0006	.9	.16	.22	.427	.68	111	.47	118	2.8	.9	.2
.0006	28.8	.16	.22	.427	.10	1070	192	145	123	7.4	1.6
.0003	34.1	.17	----	----	.08	940	59	113	90	8.3	.7
.0016	17.4	.33	.50	.555	.09	1970	460	275	240	7.2	1.9
.0019	14.7	.35	----	----	.09	1220	88	210	169	5.8	.5
.0034	20.7	48	120	1.735	.09	267	38.8	86	72	3.1	.5
.0050	1.0	.70	6.0	1.430	.84	87	2.4	99	4.7	.9	.5
.0050	42.7	.70	6.0	1.430	.05	430	87	106	99	4.1	.9
.0063	.2	.14	.38	1.258	.67	277	1.50	333	10.0	.8	.2
.0026	.6	.42	.86	.820	.67	46.5	.45	92	4.0	.5	.1
.0040	.5	.40	----	----	.55	137	1.26	168	12.6	.8	.1
.0038	.5	.50	1.1	.985	.50	400	2.0	295	20.0	1.4	.1
.0060	1.0	.95	----	----	.55	31.0	.85	63	8.5	.5	.1
.0076	66.6	.58	1.2	.870	.03	545	75	81	76	6.7	6.0

TABLE 11.—*Summary*

Station No.	Station	Hydraulic exponents m	Hydraulic exponents f	Hydraulic exponents b	Minimum variance case [1]	Log cycles of Q on plot	Amount of scatter on hydraulic geometry plots: Velocity (percent)	Depth (percent)	Width (percent)
83	Huerfano River below Huerfano Valley Dam, Colo.—low flows.	.30	.29	.41	B/C	2.4	14/23	48/14	32/24
84	Huerfano River below Huerfano Valley Dam, Colo.—high flows.	.40	.61	.00	A	.8	7/8	10/8	11/8
85	Arkansas River near Coolidge, Kansas—low flows.	.15	.18	.69	B/C	1.9	31/15	37/27	50/44
86	Arkansas River near Coolidge, Kansas—high flows.	.30	.67	.03	A	1.4	17/10	30/14	10/32
87	Blacks Fork near Little America, Wyo	.67	.28	.05	B	1.9	15/12	10/13	5/6
88	Blacks Fork near Lyman, Wyo	.30	.41	.29	B/C	1.7	15/13	30/26	25/28
89	Blacks Fork near Millburne, Wyo	.45	.23	.31	B/C	1.6	26/32	19/20	44/22
90	Wind River near Crowheat, Wyo	.44	.54	.03	A	.7	10/10	10/11	4/4
91	Muddy Creek near Shoshoni, Wyo	.36	.61	.05	B/R	1.1	23/21	25/22	7/6
92	Powder River at Arvada, Wyo	.46	.50	.07	B	1.4	20/16	21/15	23/23
93	South Fork Powder River near Kaycee, Wyo	.38	.15	.47	B/C	2.0	22/25	62/39	58/43
94	Sweetwater River near Alcova, Wyo	.36	.58	.07	B/C	1.2	14/14	21/16	6/6
95	Cheyenne River near Spencer, Wyo	.27	.28	.45	B/C	2.4	82/30	27/34	84/57
96	Lance Creek at Spencer, Wyo	.16	.31	.55	B/C	3.8	88/42	67/30	70/42
97	Belle Fourche River below Moorcroft, Wyo	.17	.37	.46	C	4.1	80/32	120/44	62/34
98	Virgin River at Virgin, Utah	.25	.13	.61	B/C	1.1	28/27	38/30	34/26
99	Summit Creek near Summit, Utah	.68	.22	.12	B	1.2	20/21	12/16	15/8
100	Coal Creek near Cedar City, Utah	.49	.25	.24	B/C	1.9	35/23	22/25	55/23
101	Santa Clara River near Santa Clara, Utah	.47	.14	.41	B/C	1.6	22/25	30/20	27/17
102	Sevier River near Circleville, Utah	.47	.41	.12	B	1.3	13/9	6/7	5/7
103	Sevier River near Lynndyl, Utah—300–400 ft below gage.	.41	.56	.05	B	1.5	11/10	13/16	14/4
104	Sevier River near Lynndyl, Utah—200-300 ft above gage.	.48	.40	.13	B/C	1.3	35/26	38/30	19/10
105	Escalante River near Escalante, Utah	.30	.32	.41	C	2.0	26/29	40/32	47/30
106	Pine Creek near Escalante, Utah	.47	.37	.19	B/C	1.7	32/39	42/24	37/34
107	San Juan River near Bluff, Utah	.43	.55	.04	B	1.4	40/26	22/24	3/6
108	Dirty Devil River above Poison Spring Wash near Hanksville, Utah.	.23	.30	.46	B/C	2.9	26/33	25/24	40/47
109	Huntington Creek near Huntington, Utah	.60	.31	.08	B	1.0	9/10	14/9	2/7
110	Saleratus Wash at Green River, Utah	.24	.29	.50	C	3.4	30/42	32/24	83/32
111	Green River at Green River, Utah	.58	.39	.03	A	1.0	6/7	10/7	1/4
112	White River below Tabbyune Creek near Soldiers Summit, Utah.	.46	.33	.21	B/C	1.6	20/26	40/26	15/24
113	Green River near Ouray, Utah	.36	.46	.19	B/C	1.4	21/21	37/20	15/13
114	Duchesne River at Myton, Utah	.57	.42	.03	A	.9	15/12	19/12	6/7
115	Tonto Creek above Gun Creek near Roosevelt, Ariz.	.47	.50	.02	A	1.5	18/16	24/25	16/6
116	San Pedro River at Winkelman, Ariz	.43	.46	.12	B/C	1.7	48/18	23/30	44/20
117	Verde River below Tangle Creek, Ariz	.57	.36	.07	B/R	.9	26/10	11/20	7/6
118	Gila River at Kelvin, Ariz	.25	.28	.49	B/C	.8	10/12	20/17	32/25
119	Gila River at head of Safford Valley near Solomon, Ariz—all flows.	.38	.38	.24	B/C	2.6	23/15	18/25	30/19
120	Gila River at Safford Valley near Solomon, Ariz.—intermediate flows.	.54	.44	.01	A	.9	20/12	21/12	15/13
121	Gila River at Calva, Ariz.—intermediate flows	.42	.53	.05	B	1.7	13/18	18/11	17/22
122	Gila River at Calva, Ariz.—high flows	.22	.67	.13	B	.6	15/8	10/12	3/6
123	Gila River near Clifton, Ariz	.38	.56	.09	B/R	1.4	19/18	20/24	21/22
124	Colorado River near Grand Canyon, Ariz	.52	.44	.04	R	1.0	3/4	5/4	2/2
125	Virgin River at Littlefield, Ariz	.38	.22	.42	B/C	1.6	31/24	82/26	32/32
126	Humboldt River near Argenta, Nev.—low flows.	.22	.32	.46	C	2.7	80/40	47/28	62/42
127	Humboldt River near Argenta, Nev.—high flows.	.32	.64	.06	B	1.2	12/10	13/15	12/7
128	Humboldt River at Comus, Nev.—low flows	.16	.29	.55	C	2.8	110/53	48/49	90/45
129	Humboldt River at Comus, Nev.—high flows	.28	.64	.08	B	1.6	13/10	16/20	9/13
130	Humboldt River at Palisade, Nev	.28	.37	.35	B/C	1.5	28/31	38/18	18/15
131	Humboldt River near Elko, Nev	.26	.69	.09	B	1.4	20/24	25/26	12/7
132	Cajon Creek near Keenbrook, Calif	.37	.23	.40	B/C	1.5	29/28	45/33	26/23
133	Santa Margarita River at Ysidora, Calif.—0–200 ft above gage.	.12	.33	.56	B/C	3.0	80/50	78/36	40/34
134	Santa Margarita River at Ysidore, Calif.—300–500 ft below gage.	.26	.29	.48	B/C	2.3	25/18	40/30	52/41
135	San Onofre Creek near San Onofre, Calif	.30	.59	.11	B	1.8	22/18	28/13	10/16
136	San Luis Rey River near Bonsall, Calif	.22	.25	.56	B/C	3.3	46/44	50/20	60/40
137	San Luis Rey River at Monserate Narrows near Pala, Calif.	.18	.13	.68	B/C	2.0	63/35	110/42	75/43
138	Warm Creek near San Bernardino, Calif	.34	.60	.04	B	1.9	19/12	14/22	13/8
139	Santa Ana River near Mentone, Calif.—low flows.	.14	.10	.74	B/C	1.9	57/38	32/21	60/40
140	Santa Ana River near Mentone, Calif.—medium flows.	.42	.38	.23	B/C	1.8	82/28	58/34	28/33
141	Wilson River near Tillamook, Oreg	.47	.52	.04	B	1.8	32/12	10/28	7/7
142	White River below Tygh Valley, Oreg	.38	.55	.05	B	1.0	15/17	25/9	5/4
143	White River near Government Camp, Oreg	.46	.47	.06	B	.9	22/7	9/16	4/6
144	Flynn Creek near Salado, Oreg	.34	.45	.24	B/C	2.1	62/47	57/27	42/30
145	Deer Creek near Salado, Oreg	.45	.40	.16	B	2.1	82/43	71/35	24/46
146	Mo'alla River near Canby, Oreg	.68	.29	.03	A	1.2	13/20	25/10	5/6
147	Nehalem River near Foss, Oreg	.43	.48	.10	B	2.0	15/19	22/14	5/7
148	Nestucca River near Beaver, Oreg	.49	.47	.06	B	1.0	5/6	6/7	1/5
149	Stehekin River at Stehekin, Wash	.72	.24	.04	B	.9	8/7	2/4	6/4
150	Spokane River above Liberty Bridge near Otis Orchards, Wash.	.59	.18	.23	B	.9	5/5	2/7	2/6
151	Okanogan River at Malott, Wash	.62	.33	.05	B	1.2	20/11	8/18	4/5
152	Little Spokane River at Dartford, Wash	.44	.45	.11	B	.7	4/4	5/7	3/6
153	Kettle River near Ferry, Wash	.36	.60	.04	B	1.0	5/4	6/5	2/3
154	White River near Plain, Wash	.29	.71	.01	A	1.0	9/15	13/10	2/2
155	Naches River below Tieton River near Naches, Wash.	.59	.34	.07	B	1.1	10/12	12/9	2/4

See footnotes at end of table.

of data—Continued

Slope S (ft/ft)	Average bank angle $\bar{\theta}$ (degrees)	d_{50} (mm)	d_{84} (mm)	Sorting So	$\frac{b_1}{b_1+f_1}$	A_{max} (ft²)	A_{min} (ft²)	W_{max} (ft)	W_{min} (ft)	D_{max} (ft)	D_{min} (ft)
.0113	.6	.35	.68	1.100	.71	83	7.1	123	21.0	.7	.3
.0113	.0	.35	.70	1.111	.00	261	83	123	123	2.1	.7
.0013	.7	.68	----	----	.65	252	8.0	200	21.0	1.3	.4
.0013	23.1	.68	----	----	.05	700	252	210	200	3.3	1.3
.0008	17.5	18	30	.975	.15	670	95	117	86	5.7	1.1
.0008	1.5	.35	.53	.470	.57	142	1.24	106	7.0	1.3	.2
.0140	2.6	46	70	.573	.53	220	.93	100	5.3	2.2	.2
.0033	25.4	35	43	.301	.09	1330	301	188	165	7.1	1.8
.0041	41.8	.25	----	----	.06	51	10.7	30	27	1.7	.4
.0009	9.3	.18	----	----	.14	113	43.7	84	74	1.3	.6
.0019	.5	.55	----	----	.71	26.0	.39	78	3.9	.3	.1
.0011	32.9	.97	2.6	.914	.06	200	52.5	76	70	2.6	.8
.0018	.7	.80	----	----	.69	74	.92	110	5.2	.7	.2
.0014	.5	.30	----	----	.68	31.5	.48	84	4.8	.4	.1
.0006	11.1	3.1	7.6	1.566	.32	387	1.15	68	10.4	5.7	.1
.0080	.9	.33	----	----	.47	175	1.79	157	18.0	1.1	.1
.0450	34.3	24	53	.820	.15	10.5	1.11	9.8	7.0	1.1	.2
.0170	4.5	12	38	1.248	.54	26.0	.62	27	3.5	1.0	.2
.0076	1.9	.73	2.1	1.041	.66	3.35	.29	14	2.9	.2	.1
.0006	22.7	.41	1.0	.699	.21	180	13	43	24	4.2	.5
.0006	24.2	.22	10.0	2.244	.10	317	52	84	70	3.8	.8
.0006	17.4	.22	9.7	2.243	.15	190	58	70	58	2.7	1.0
.0080	2.3	14	21.5	.447	.54	6.4	.46	22	4.6	.4	.1
.0213	23.4	----	----	----	.32	81	2.2	23	7.2	3.5	.3
.0019	25.6	.30	----	----	.09	1500	222	190	162	7.9	1.4
.0024	1.0	.083	.25	1.107	.52	77	1.0	99	10.0	.8	.1
.0160	14.7	6.2	12.1	1.845	.17	85	7.6	42	28	2.0	.3
.0020	5.9	.062	.75	1.875	.37	1400	1.9	172	15	8.1	.1
.0009	30.8	.074	.16	.824	.06	3500	1000	335	310	10.4	3.2
.0060	4.4	2.7	8.9	2.497	.35	48	.91	40	9.9	1.2	.09
.0002	2.6	.20	----	----	.45	4100	31	445	49	9.2	.6
.0029	46.4	----	----	----	.10	545	172	79	70	6.9	2.5
.0035	17.0	1.05	15	2.391	.06	460	126	182	170	2.5	.7
.0030	11.0	.66	----	----	.21	600	98	125	84	4.8	1.2
.0047	31.0	23	87	1.398	.13	440	40	70	51	6.3	.8
.0015	1.5	.42	----	----	.57	96	.77	88	5.7	1.1	.1
.0019	3.3	.48	----	----	.43	2600	6.2	310	22	8.4	.3
.0019	16.8	.48	----	----	.08	480	113	151	135	3.2	.8
.0017	30.2	----	----	----	.08	180	56	71	65	2.5	.9
.0017	39.1	----	----	----	.13	705	180	86	71	8.2	2.5
.0017	36.3	----	----	----	.11	760	84	92	72	8.3	1.2
.0001	50.4	.28	----	----	.07	11000	880	315	262	35.0	3.4
.0021	.4	.27	----	----	.74	101	3.7	160	14	.6	.3
.0006	2.7	1.8	15	2.427	.49	36.2	.93	41	6.7	.9	.1
.0006	29.4	6.6	20	1.865	.10	660	62	101	80	6.5	.8
.0004	.7	.66	4.8	1.590	.75	19.0	.10	54	1.0	.4	.1
.0004	24.8	.90	5.4	1.642	.12	500	19	80	54	6.3	.4
.0012	2.5	.53	1.2	.925	.41	147	1.2	87	12	1.7	.1
.0015	36.4	.60	2.2	1.491	.11	940	157	106	86	8.9	1.8
.0246	3.5	1.6	12.7	1.793	.42	160	.76	76	7.6	2.1	.1
.0048	1.7	.27	.71	1.086	.50	600	3.8	206	16	2.9	.2
.0048	1.1	.16	.38	1.146	.65	104	.50	102	3.1	1.0	.2
.0063	17.9	.58	4.5	1.942	.17	166	7.3	49	29	3.4	.3
.0037	1.2	.20	.44	.911	.61	270	.21	160	2.1	1.7	.1
.0030	.5	.36	.77	.806	.73	31.0	.17	81	1.7	.4	.1
.0038	29.5	.78	3.3	1.371	.09	202	17.7	59	48	3.4	.4
.0189	1.9	43	149	1.754	.71	5.1	.14	17	1.4	.3	.1
.0189	8.3	15	98	2.727	.31	75	5.1	41	18	1.8	.3
.0027	27.7	100	240	1.121	.11	2400	213	185	140	13.0	1.5
.0096	17.4	----	----	----	.12	322	81	96	80	3.4	1.0
.0142	20.6	13	62	2.125	.14	177	44	64	53	2.8	.8
.0300	14.3	2.8	11.7	1.706	.37	96	.81	30	5.2	3.2	.2
.0217	31.0	32	88	1.164	.20	59	2.0	19	9.8	3.0	.2
.0019	31.9	36	75	.981	.09	1220	290	157	138	7.8	2.1
.0008	13.1	45	110	.910	.20	4200	72	254	115	16.5	.6
.0019	23.8	23	65	1.220	.08	1120	68	149	117	7.5	.6
.0060	21.0	77	244	1.477	.21	1250	390	144	113	8.7	3.5
.0020	1.8	33	115	1.288	.56	2200	3.0	370	9.0	5.9	.3
.0001	24.9	59	150	1.869	.10	3950	257	260	199	15.2	1.3
.0019	38.0	63	153	1.301	.08	302	29.0	63	52	4.8	.6
.0009	24.4	56	157	1.000	.09	2200	340	227	191	9.7	1.8
.0007	44.6	13	41	2.136	.07	1210	112	117	98	10.3	1.1
.0049	7.9	86	147	.608	.18	540	67	147	101	3.7	.7

TABLE 11.—*Summary*

Station No.	Station	Hydraulic exponents			Minimum variance case [1]	Log cycles of Q on plot	Amount of scatter on hydraulic geometry plots		
		m	f	b			Velocity (percent)	Depth (percent)	Width (percent)
156	Yakima River at Cle Elum, Wash	0.37	0.60	0.02	A	0.7	3/5	7/5	1/2
157	Klickitat River near Glenwood, Wash	.53	.43	.03	A	1.0	9/9	9/10	3/7
158	Tucannon River near Starbuck, Wash	.33	.60	.07	B	.9	10/7	7/6	5/7
159	Methow River near Pateros, Wash	.49	.44	.05	B	1.2	3/7	5/4	2/2
160	American River near Nile, Wash	.51	.38	.10	B	2.0	10/5	6/12	4/6
161	Skagit River near Mt. Vernon, Wash	.35	.61	.04	B	1.0	5/6	8/7	2/3
162	Green River near Auburn, Wash	.42	.54	.04	B	1.3	3/8	6/5	1/4
163	Carbon River at Fairfax, Wash	.54	.41	.05	B	1.5	30/10	13/15	5/6
164	Chehalis River near Grand Mound, Wash	.67	.31	.02	A	1.0	3/4	7/4	3/4
165	Snoqualmie River near Carnation, Wash	.81	.15	.02	A	1.2	7/7	5/10	1/2

[1] A = approximately vertical banks;
B = banks firm but not vertical;
C = loose, noncohesive banks;
R = rock banks;
B/C, B/R, C/R = one bank of each type indicated (for example, B/C = one bank firm but not vertical, the other bank noncohesive).

of data—Continued

Slope S (ft/ft)	Average bank angle θ (degrees)	d_{50} (mm)	d_{84} (mm)	Sorting So	$\frac{b_1}{b_1+f_1}$	A_{max} (ft^2)	A_{min} (ft^2)	W_{max} (ft)	W_{min} (ft)	D_{max} (ft)	D_{min} (ft)
0.0030	22.0	37	66	1.393	0.04	1190	270	267	250	4.5	1.1
.0059	15.5	100	170	.723	.12	370	23.5	96	70	3.9	.3
.0064	37.2	24	37	.544	.12	435	53	67	52	6.5	1.0
.0036	30.3	58	172	1.653	.08	2200	245	202	169	10.9	1.4
.0240	15.2	88	217	1.033	.25	258	36	65	40	4.0	.9
.0002	23.1	.65	2.6	1.286	.06	14600	940	640	540	22.8	1.7
.0005	30.0	100	140	1.176	.07	1570	108	174	145	9.0	.7
.0120	35.2	90	130	1.029	.05	475	86	110	100	4.3	.9
.0004	37.5	104	160	.893	.08	2950	635	241	216	12.2	2.9
.0011	31.8	95	----	----	.09	3600	770	260	225	13.8	3.4