Basic Concepts in Three-Part Quantitative Assessments of Undiscovered Mineral Resources

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Since 1975, mineral resource assessments have been made for over 27 areas covering 5 × 10^6 km^2 at various scales using what is now called the three-part form of quantitative assessment. In these assessments, (1) areas are delineated according to the types of deposits permitted by the geology, (2) the amount of metal and some ore characteristics are estimated using grade and tonnage models, and (3) the number of undiscovered deposits of each type is estimated.

Permissive boundaries are drawn for one or more deposit types such that the probability of a deposit lying outside the boundary is negligible, that is, less than 1 in 100,000 to 1,000,000.

Grade and tonnage models combined with estimates of the number of deposits are the fundamental means of translating geologists' resource assessments into a language that economists can use.

Estimates of the number of deposits explicitly represent the probability (or degree of belief) that some fixed but unknown number of undiscovered deposits exist in the delineated tracts. Estimates are by deposit type and must be consistent with the grade and tonnage model. Other guidelines for these estimates include (1) frequency of deposits from well-explored areas, (2) local deposit extrapolations, (3) counting and assigning probabilities to anomalies and occurrences, (4) process constraints, (5) relative frequencies of related deposit types, and (6) area spatial limits. In most cases, estimates are made subjectively, as they are in meteorology, gambling, and geologic interpretations.

In three-part assessments, the estimates are internally consistent because delineated tracts are consistent with descriptive models, grade and tonnage models are consistent with descriptive models, as well as with known deposits in the area, and estimates of number of deposits are consistent with grade and tonnage models. All available information is used in the assessment, and uncertainty is explicitly represented.

Key words:
Mineral resource assessment
Mineral deposit models
Quantitative assessment
Three-part assessments

Introduction

The purpose of quantitative mineral resource assessments is to provide predictions of the consequences of alternative courses of action with respect to minerals on tracts of land. Using resource assessments we can plan exploration, consider alternate uses of land, plan economic development, and estimate the availability of minerals under different conditions. Typically, the main problem is predicting undiscovered mineral deposits. Because the deposits are undiscovered, uncertainty is an integral part of the problem.

The first attempt to effectively deal with this problem was an article by Allais (1957) on the possible returns of exploration for minerals in the Algerian Sahara. More recently, many articles using a wide variety of methods and forms of quantitative mineral resource assessment have been published (see reviews in Singer and Mosier, 1981b; Harris, 1984; Shulman and others, 1992).

In this article I describe a form of quantitative mineral resource assessment of nonfuel mineral commodities that has evolved and has been used in over 27 different assessments. The purpose of this article is to provide readers with some of the basic concepts that underlie this form of assessment and that may prove useful in other forms of quantitative assessment.

What is now called the three-part form of quantitative assessment has been applied by the U.S. Geological Survey since 1975. Its original purpose was to provide quantitative resource information in a form consistent with an economic analysis so that mineral resource values could be compared with values derived from other competing uses of land (Singer, 1975). Early applications of this form of assessment were used to respond to land classification problems in Alaska. Later, the ability to combine the probabilistic parts through simulation was added (Root and others, 1992).

Using the three-part quantitative assessment, approximately 5 × 10^6 km^2 have been assessed at various scales, including areas with diverse geologic environments and varied levels of information, in North, Central, and South America. In three-part assessments (fig. 1), (1) areas are delineated according to the types of deposits permitted by the geology, (2) the amount of metal and some ore characteristics are estimated using grade and tonnage models, and (3) the number of undiscovered deposits of each type is estimated.

It is necessary to have a geologic map and it is desirable to have mineral occurrence, geophysical, exploration, and geochemical information to delineate areas or domains that are permissive for different deposit types (fig. 2). This information must be integrated with information about the types of mineral deposits that occur in each geologic setting.

Grade and tonnage models of mineral deposits are useful in quantitative resource assessments and exploration planning. Having some idea of the possible values of alternative kinds of deposits that might be sought is critical to good exploration planning. According to Sangster (1986), grade and tonnage models are the Rosetta stone that, when combined with estimates of the number of deposits, translates geologists' resource assessments into a language that economists can understand. In three-part assessments, previously constructed grade and tonnage models are typically used unless local deposits are significantly different than the general model.

Estimates of the number of undiscovered deposits are presented in a probabilistic manner to convey the uncertainty associated with the estimates. An important consideration in making these estimates is the link between the estimates of the number of deposits and the grade and tonnage models: The two must be consistent for the assessment to be realistic.

Deposit models represent the glue that bonds together diverse information on geology, mineral occurrence, geochemistry, and geophysics that is used in mineral exploration and resource assessments (fig. 2). The ability to plan exploration and to make better resource assessments depends directly on the quality of the deposit models. In the following sections, I discuss some of the concepts that are important in three-part assessments and in deposit models as they apply to these assessments.

Mineral Deposit Models

In most published three-part assessments, two kinds of models have been relied upon—descriptive, and grade and tonnage. Examples of a third kind of model that represents the number of deposits per unit area have recently appeared (Bliss and others, 1987; Bliss, 1992b; Singer, in press b). Probably the most important part of building a mineral deposit model is the planning stage in which consideration of the purpose and possible uses of the model determines its character. Ideally, deposit models provide the necessary and sufficient information to discriminate possible mineralized environments from barren environments, types of known deposits from each other, and mineral deposits from mineral occurrences. In three-part assessments (Singer and Cox, 1988), deposit models are used, in the delineation part of the assessment, to classify mineralized and barren environments and to classify types of known deposits, whereas mineral deposits are distinguished from mineral occurrences in the

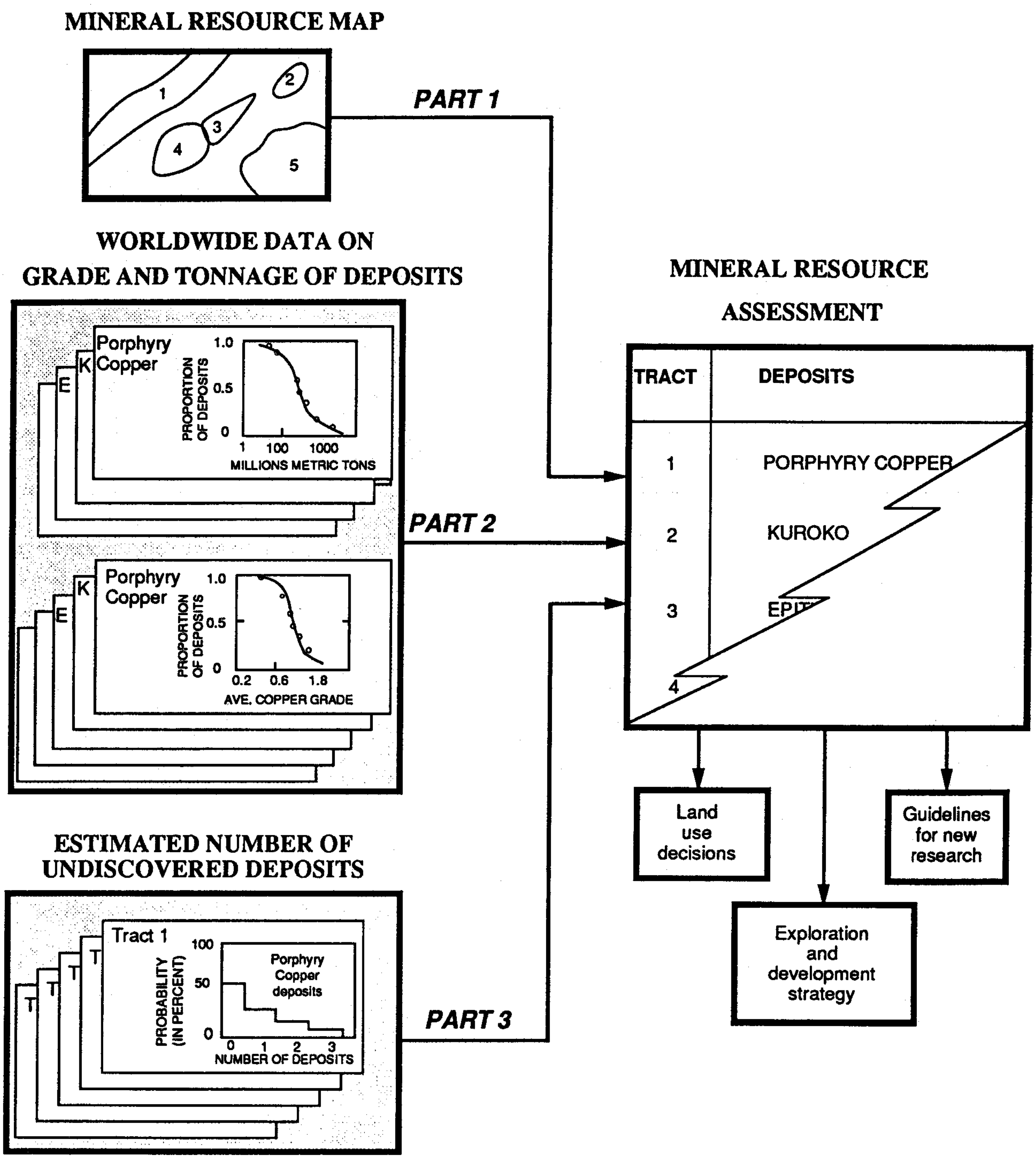

Figure 1. Three parts make up a quantitative mineral resource assessment, which has many applications.

number of deposits estimation part of the assessment. A wide variety of geoscience information from the region of interest is used for these tasks (fig. 2).

The keystone to combining this diverse information is the mineral deposit model. Documented deposit models (Cox and Singer, 1986; Orris and Bliss, 1991; Bliss, 1992a) allow geologists to link deposit types to geologic environments. A mineral deposit is a mineral occurrence of sufficient size and grade that it might, under the most favorable circumstances, be considered to have economic potential (Cox and others, 1986). This is the target population in three-part assessments. Because every mineral deposit is different from every other in some way, models have to progress beyond the purely descriptive to represent more than single deposits. Deposits sharing a relatively wide variety and large number of attributes come

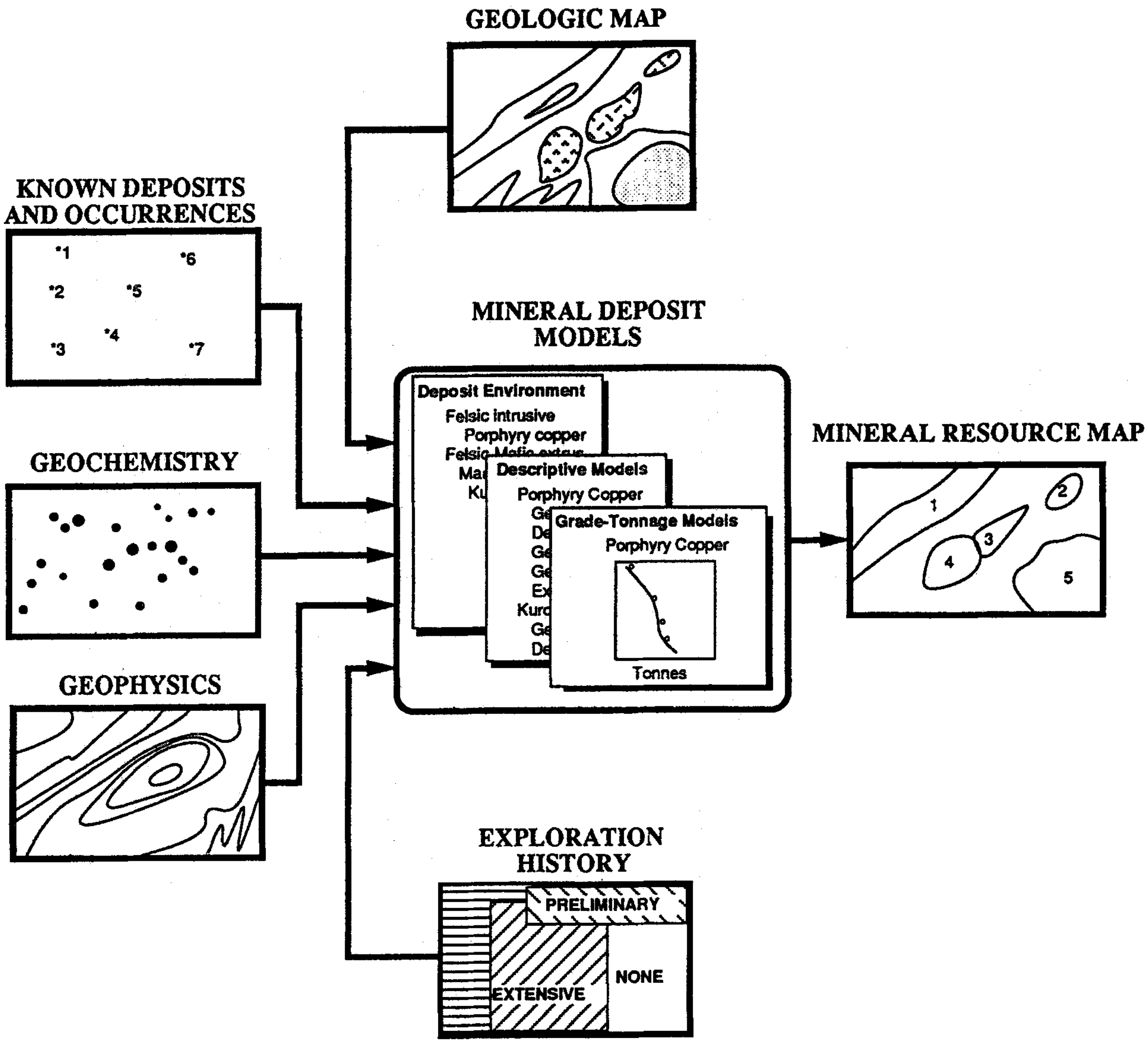

Figure 2. Diagram showing how deposit models integrate diverse information used in mineral resource assessments.

to be characterized as a "type," and a model representing that type evolves.

Descriptive models have two parts. The first part describes the geologic environments in which the deposits are found; the second part describes the identifying characteristics of the deposits. The first part plays a primary role in the delineation process in that it describes the general setting of the deposit type. The second part helps classify known deposits and occurrences into types, which aids the delineation process. In some cases, geologic environments not indicated on geologic maps are identified by the types of known deposits and occurrences. The organization of the models constitutes a classification of deposits. This arrangement provides ready access to the models by focusing on host-rock lithology and tectonic setting, the features most easily obtained from a geologic map.

Grade and tonnage models have the form of frequency distributions of tonnages and average grades of well-explored deposits of each type. They serve as models for grades and tonnages of undiscovered deposits of the same type in geologically similar settings. Although this type of modeling has been ongoing for about 15 years, the recent publication of 69 grade and tonnage models (Cox and Singer, 1986; Bliss, 1992a) is the largest collection of models available for resource assessments. These models are presented as cumulative frequency graphs and as summary statistics. A best-fit lognormal curve, based on the mean and standard deviation of the data, is also provided. For each deposit type, these models help define a deposit, as opposed to a mineral occurrence or a weak manifestation of an ore-forming process.

Construction of grade and tonnage models involves multiple steps (Singer, in press a), the first of which is the identification of a group of well-explored deposits that are believed to belong to the mineral deposit type being modeled. A descriptive model is commonly prepared as well; the attributes of each deposit in the group are compared with the descriptive model to ensure that all are of the same type. Data gathered for each deposit include average grades of each metal or mineral commodity of possible economic interest and the associated tonnage based on the total production, reserves, and resources at the lowest possible cutoff grade. All further references to tonnage follow this definition. These data represent an estimate of the endowment of each known deposit. The final model represents the endowment of all undiscovered deposits. Statistics from published grade and tonnage models indicate that most have a lognormal distribution of tonnage. Moreover, significant correlations between tonnage and grade of the major commodity are uncommon (table 1). Such information about frequency distributions and correlations is required in the construction of some simulators.

In practice, grades and tonnages for different deposits of the same type are seldom reported at the same cutoff grade. In fact, cutoff grades are reported only infrequently. A second consideration at the data-gathering stage is the question of what the sampling unit should be (Singer, in press a). Grade and tonnage data are available to varying degrees for districts, deposits, mines, and shafts. In many cases old production data are available for some deposits and recent resource estimates are available for other deposits. The most common error in constructing grade and tonnage models is mixing old production data from some deposits with resource data from other deposits. It is important that all data used in the model represent the same sampling unit. Mixing data from deposits and districts or mixing old production and recent resource estimates usually produces bimodal or at least non-lognormal distributions and may introduce correlations among the variables that are artifacts of the mixed sampling units. Models constructed using data from mixed sampling units are of questionable value because the frequencies of tonnage and grade observed are directly related to the proportion of deposits from each sampling unit and are unlikely to be representative of the proportion in the undiscovered deposits being estimated in an assessment.

The application of these models to resource assessments helps to identify how the models should be augmented. Failure to distinguish between the probability of existence of an attribute and the probability of the conjunction of attributes leads to problems in the application of the models. For example, it is possible that each attribute exists in most deposits of a type and, at the same time, only a small number, or none, of the deposits have all of these attributes. Quantifying mineral deposit attributes is the necessary and sufficient next step in statistically classifying known deposits by type (Chung, Jefferson, and Singer, 1992). The same information is necessary but not sufficient to discriminate barren from mineralized environments; quantifying the attributes of barren environments is also necessary for this task. For the models of the number of deposits per unit area and for the attempts to quantify deposit attributes to be useful in three-part assessments, they must be constructed so that they are consistent with the present descriptive and grade and tonnage models. Without this, the resulting resource assessments are internally inconsistent.

Delineation

Areas or domains that may contain particular deposit types (Lasky, 1948) are delineated by analogy with similar geologic settings containing known deposits elsewhere. To construct the boundaries, it is necessary to have a geologic map and it is desirable to have mineral occurrence, geophysical, exploration, and geochemical information (fig. 2). This information must be integrated with information about the geologic environments expected for different types of mineral deposits.

One delineation strategy is to move boundaries outward from known deposits. This might be considered the delineation of favorable areas. In three-part assessments, we try to delineate permissive areas. Although favorable areas are a subset of permissive areas, they represent very different concepts. Their boundaries will coincide only if exploration coverage is very thorough and completely effective—a fairly unusual situation. In addition, delineations of favorable areas are frequently applied in different ways by different people because of the difficulty of defining a commonly acceptable operational rule. Known deposits and occurrences serve to identify and expand permissive tracts, not constrain them.

For consistency, areas are delineated where the geology permits the existence of deposits of one or more specified types. These areas, called permissive tracts, are based on geologic criteria derived from deposit models that are themselves based on studies of known deposits within and outside the study area. Permissive boundaries are defined such that the probability of deposits of the type delineated occurring outside the boundary is negligible, that is, less than 1 in 100,000 to 1,000,000. Using this definition, it is possible to subdivide a permissive tract into two or more parts that have different kinds of information or possibly different numbers of undiscovered deposits, such as the number of epithermal gold veins in Costa Rica (Singer and others, 1987) or porphyry

Table 1. Statistics of tests of lognormality of tonnages and correlations of tonnage and grade of main commodity.

Deposit type	Tonnage skewness	Tonnage peakedness	Kolmogorov–Smirnov (Lilliefors) probability	Number of deposits	Tonnes versus main commodity correlation coefficient
Algoma & Superior Fe	0.46	1.07	0.131	66	−0.168
Bedded barite	−0.2	−0.98	0.634	25	−0.019
Besshi massive sulfide	0.77	−0.41	0.012	44	0.123
Carbonatite	−0.55	−0.24	0.909	20	−0.222
Climax Mo	−0.89	0.17	0.688	9	−0.013
Comstock epithermal vein	−0.16	0.21	0.539	41	0.041
Creede epithermal vein	−0.21	−0.68	0.840	27	−0.373
Cu skarn	−0.02	−0.41	1.000	64	−0.303
Cyprus massive sulfide	0.14	0.74	0.513	49	−0.089
Distal disseminated Ag-Au	−0.51	−0.57	0.859	10	−0.553
Dunitic Ni-Cu	−0.32	−0.83	0.362	22	−0.54**
Epithermal Mn	−0.24	−0.02	0.979	59	−0.276
Epithermal Qtz alunite Au	−0.35	−0.69	0.985	9	−0.160
Homestake Au	−0.04	0.27	0.189	118	−0.088
Hot-spring Au	−0.58	0.36	0.821	17	0.379
Hot-spring Hg	−0.06	−0.98	0.625	20	−0.042
Iron skarn	−0.01	−0.2	0.602	169	−0.070
Karst Bauxite	−0.32	−0.33	0.088	41	0.123
Komatiitic-Ni-Cu	0.23	−0.91	0.053	31	−0.47**
Kuroko massive sulfide	0.09	−0.45	0.474	432	−0.168**
Kuroko massive sulfide, Sierran	0.47	−0.69	0.240	23	−0.030
Lateritic Bauxite	−0.06	−0.59	0.353	122	−0.047
Lateritic Ni	0.17	0.85	0.360	71	−0.31**
Low-sulfide Au-Qtz veins	0.15	−0.38	0.457	333	−0.298**
Low-sulfide Au-Qtz veins, Chugach	−0.14	−0.7	0.589	29	−0.120
Phosphate, upwelling	0.09	−0.22	0.163	60	−0.148
Phosphate, warm-current	0.43	−0.48	0.657	18	−0.536
Placer Au-PGE	−0.4	−0.93**	0.597	65	−0.347**
Placer PGE-Au	0.03	−0.56	0.068	83	−0.423**
Podiform chromite, major	0.47**	−0.07	0.020	174	−0.014
Podiform chromite, minor	0.55**	0.13	0.025	435	−0.254**
Polymetallic replacement	0.05	−0.72	1.000	52	0.160
Polymetallic vein	0.25	−0.93**	0.140	75	−0.277
Porphyry Cu	−0.16	0.32	1.000	209	0.108
Porphyry Cu, skarn-related	0.5	−1.04	0.485	18	0.078
Porphyry Cu-Au	−0.06	−0.24	0.339	40	0.099
Porphyry Cu-Mo	0.56	0.18	0.501	16	0.436
Porphyry Mo, low-F	0.03	−0.17	1.000	33	−0.229
Replacement Mn	0.42	−0.96	0.188	37	−0.257
Replacement Sn	−0.4	−0.55	1.000	6	0.259
Rhyolite-hosted Sn	0.04	−0.11	0.435	132	0.359**
Sado epithermal vein	0.37	−0.46	1.000	20	−0.132
Sandstone-hosted Pb-Zn	−0.01	−0.4	0.537	20	−0.111
SE Missouri Pb-Zn & Appalachian Zn	−0.13	−0.96	1.000	20	−0.055
Sediment-hosted Au	0.61	0.57	0.923	39	−0.090
Sediment-hosted Cu	−0.02	−0.56	0.571	57	−0.029
Sedimentary exhalative Zn-Pb	−0.25	−0.81	0.570	45	0.087
Sedimentary Mn	−0.08	−0.74	1.000	38	−0.113
Serpentine-hosted asbestos	−0.29	−0.06	0.310	50	−0.133
Shoreline placer Ti	−0.29	−0.19	0.243	61	−0.162
Silica-carbonate Hg	0.49	−0.48	0.382	28	0.077
Simple Sb	0.09	−0.03	0.133	81	−0.219
Simple Sb, dissemimated	0.04	−1.1	0.983	23	0.137
Sn greisen	0.2	−1.48	0.481	10	−0.353
Sn skarn	−0.09	−1.82	0.558	4	−0.808
Sn veins	−0.15	−0.9	0.037	43	−0.195
Synorogenic-synvolcanic Ni-Cu	−0.07	−0.7	0.602	32	−0.260
Unconformity U	−0.32	−0.62	0.132	36	−0.122
Volcanic-hosted magnetite	0.08	−0.75	0.538	39	−0.015
Volcanogenic Mn, Cuban	−0.74**	−0.17	0.383	93	−0.216
Volcanogenic Mn, Cyprus	0.26	−1.04	0.496	7	−0.265
Volcanogenic Mn, Franciscan	0.37	−0.07	0.444	184	−0.110
Volcanogenic Mn, Oly Pen	0.09	−0.16	0.477	17	0.210
Volcanogenic U	0.11	−0.24	0.732	21	−0.346
W skarn	−0.54	−0.11	0.759	28	−0.178

Table 1. Continued.

Deposit type	Tonnage skewness	Tonnage peakedness	Kolmogorov-Smirnov (Lilliefors) probability	Number of deposits	Tonnes versus main commodity correlation coefficient
W veins	−0.12	−1.24	0.293	16	−0.333
Zn-Pb skarn	−0.06	−0.93	0.816	34	−0.045
Total number (67 types)	3 significant	2 significant	0 significant	4350	9 significant (−8:+1)

** Significant at the 1-percent level.

copper deposits in part of Alaska (Singer and MacKevett, 1977). Other less stringent definitions of delineation boundaries have been considered, but they suffer from two flaws: They are difficult to define in a manner that can be consistently applied, and they may exclude areas that contain rare but very large deposits. The cost of the error of missing the rare large deposit results in the Bayesian trade-off decision (minimized cost of misclassification) in favor of the negligible error definition of permissive tracts.

Tracts may or may not contain known deposits. Areas are excluded from these tracts only on the basis of geology, knowledge about unsuccessful exploration, or the presence of barren overburden exceeding some predetermined thickness. Thus, the fundamental information is the geologic map and extensions of geologic units under cover as extrapolated by geologic and geophysical considerations. Information from geochemistry and known deposits and occurrences helps identify environments and, in some cases, excludes areas. For example, low-sulfide gold-quartz veins frequently have placer gold deposits associated with them; if no placer gold deposits exist, then the environment of the associated gold deposit type might not exist.

A geologic map is the primary local source of information for delineating domains and identifying which domains are permissive for different deposit types. Probably the second most important type of information is an inventory of known deposits and prospects in and near the region being assessed. Owing to incomplete deposit descriptions, it is often difficult to identify deposit types for many prospects and some deposits, but those that can be identified increase confidence in domains delineated for the deposit type. Prospects may indicate possible deposit types and place limits on what is possible elsewhere. A map distribution of deposits that can be classified by type then serves as a check on the accuracy of the delineation of tracts permissive for that type rather than as a determinant of the delineation. Geochemistry of stream sediments may suggest deposit types and aid delineation of domains for some deposit types. Geophysics contributes by identifying extensions of permissive rock units under cover and by identifying rock units in poorly mapped areas; in some cases, geophysics can identify favorable rock units, such as hydrothermally altered rocks. Both stream sediment and rock geochemistry can provide similar benefits to large regional assessments.

Domains are outlined for the possibility of the existence of one or more deposit types as inferred by analogy with deposits in similar geologic settings elsewhere. Mineral deposit models provide the means to make the links between geologic settings and deposit type. In every case, the boundaries of the domains are based first on mapped or inferred geology. Original boundaries are reduced only where it can be firmly demonstrated that a deposit type could not exist. For some deposit types, extensive exploration might provide such evidence, but for many deposit types, only close-spaced drilling can be used to exclude areas.

Designation of a tract as permissive does not imply any special favorability for the occurrence of a deposit, nor does it address the likelihood that a deposit will be discovered there if it exists. The probability of discovery of deposits involves a large number of uncertainties, such as future economic conditions, development of new exploration methods, depth and type of cover, and the determination of the explorer, all of which are typically beyond the scope of these studies.

In some cases, the scale of product or working maps requires that a certain amount of generalization takes place in the delineation. For example, in a recent analysis of Nevada's resources, skarns, which are known to occur within 2 km of plutons, were delineated with more distal pluton-related deposits, such as polymetallic replacement districts, in 10-km-wide tracts around plutons because 2-km-wide tracts would have been nearly invisible at the 1:1,000,000 publication scale. The grouping of deposit types in the delineation does not mean that deposit types will be grouped in the grade and tonnage models or in the estimates of the number of deposits.

Grades and Tonnages

A critical part of the exploration for mineral deposits and of quantitative mineral resource assessments is the estimation of the sizes of undiscovered deposits. Typically, this problem is addressed using grade and tonnage models because a major source of variation in possible sizes is accounted for by the differences in types of deposits (Cox and Singer, 1986; Bliss, 1992a).

In quantitative resource assessments, grade and tonnage models play two roles. First, they help classify the known deposits in a region and, therefore, aid in delineation. Second, they provide information about the potential value of undiscovered deposits in the assessment area and are thus the key to economic analyses of these resources (Singer and Cox, 1988). Before discussing the application of grade and tonnage models, it is desirable to address questions about the effects of economic filtering, cutoff grades, skewed tonnage distributions, and size-biased sampling.

Deposits suspected to be small or very low grade are seldom sampled well enough to be characterized in terms of grade and tonnage; therefore, one would expect that the sample of many deposit types would be truncated by economics. Effects of economic filtering should be most evident in plots of grade versus tonnage for which the combination of low grade and low tonnage should be missing. The analysis of Harris (1984) of uranium deposits in New Mexico demonstrated a positive correlation between grade and tonnage when all data were used, a negative correlation when small deposits were removed, and no correlation when economic effects of truncation were removed. For almost any conceivable distribution of grades and tonnages before economic filtering, the removal of low-grade and low-tonnage deposits because of economics would cause a negative correlation in the remaining data. If the true relationship between grade and tonnage of all mineralization (deposits and occurrences) of each type were inverse, then the effect of excluding small and low-grade deposits by an economic filter would not hide the relationship, and it might enhance its inverse nature. Negative correlations are rare in published grade and tonnage models (table 1), suggesting that economic filtering is not severe. Probably 40 percent of the deposits used in these models are, in fact, noneconomic today. For example, at least 50 percent of the deposits used in the grade and tonnage model for porphyry copper have never been developed even though most were explored more than 15 years ago. About 90 percent of the 33 porphyry molybdenum, low-fluorine deposits have never been developed. The majority of the 435 podiform chromite deposits from California and Oregon were mined only when there was a subsidy. The figures in Cox and Singer (1986) reveal examples of both small deposits and low-grade deposits.

Taylor (1985) combined the theoretical aspects of the lognormal distribution of grades within deposits with actual examples and economic analysis to show how cutoff grades can, in practice, affect average grades and tonnages of deposits. He concluded that the cutoff grade for a deposit must be near the median of the grades within a deposit to recover a reasonable proportion of the metal content in a tonnage fraction that is sufficiently large to have spatial continuity and to be minable. He also observed that many cutoff grades of mines are located at or near the median grade within deposits. Thus, although wide variability in tonnages and average grades may result from changes in cutoff grades, in practice, operators are limited to a rather narrow range of cutoff grades by economics, by the need to mine contiguous blocks, and by the consequences of dealing with the lognormal distribution of grades within deposits. Exceptions may exist, however, because of differences in mining methods that significantly affect operating costs, such as the very low cost of dredge mining and heap leaching for gold. Although further work is needed to define the relation of cutoff grade to these models, the effect of cutoff grades on grade and tonnage models may not be as pronounced as suspected, provided the mining method is the same.

Potential metal supply is dominated by the very few largest tonnage deposits of the largest deposit types (fig. 3), as shown by Singer and DeYoung (1980), who also pointed out that inverse correlations between grade and tonnage are surprisingly rare. Therefore, most low-grade deposits are not likely to have huge resources; omitting a few low-grade or small-tonnage deposits will not seriously degrade the predictions of potential supplies of most commodities. Given that many deposits included in grade and tonnage models are apparently uneconomic, the ability to add smaller deposits to most deposit type models would not seem to be of any practical value. Lower-grade deposits might be of interest if they have large tonnages and if new low-cost mining methods are possible.

It is clear from studies of petroleum exploration that larger oil fields tend to be found early in the exploration process (Arps and Roberts, 1958). If the same is true in mineral exploration, then tonnage models constructed from local data may be biased estimators of the tonnages of the remaining undiscovered deposits in the area. Singer and Mosier (1981a) showed that larger porphyry copper deposits should be found earlier than smaller deposits in a given geologic and exploration environment, but studies that test the hypothesis with actual exploration data are rare (Stanley, 1992).

Analysis of the discovery order of mercury deposits

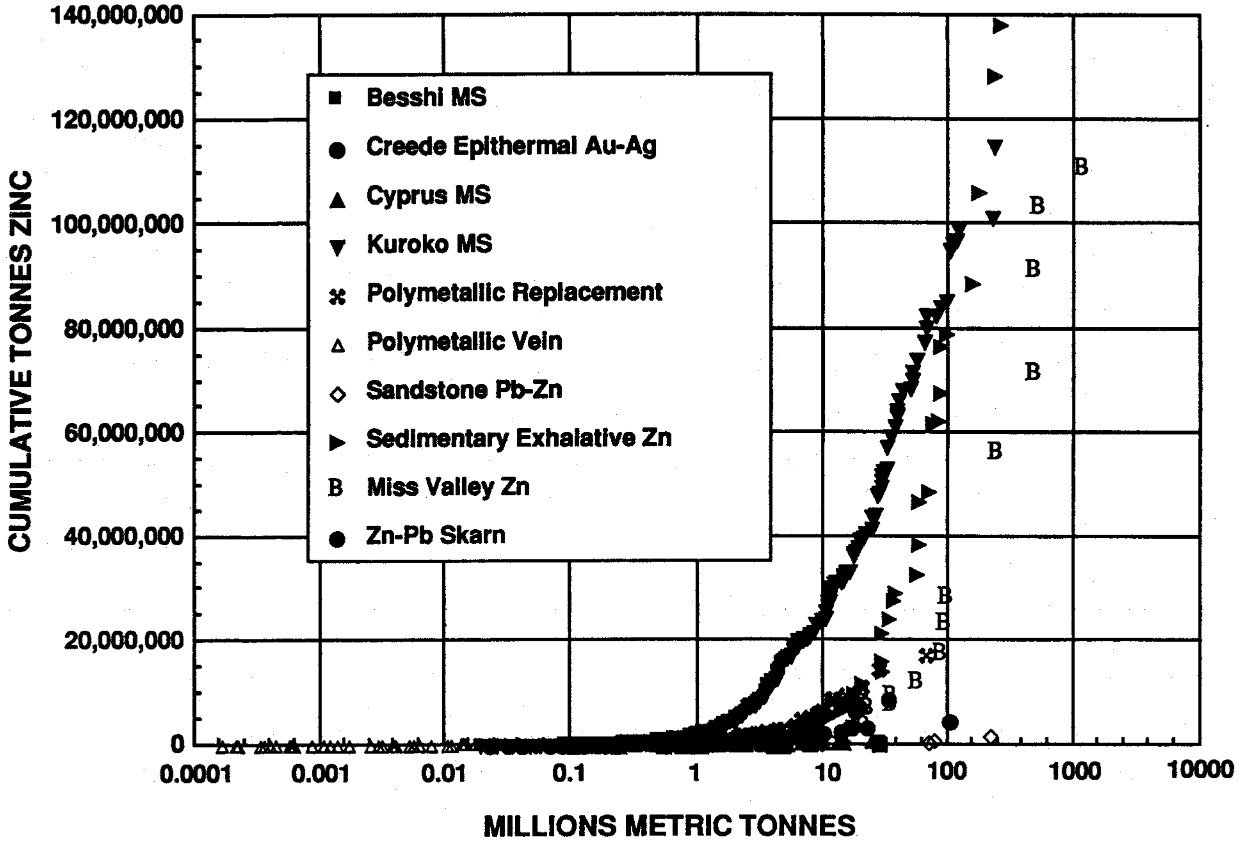

Figure 3. The largest tonnage deposits of the largest deposit types dominate potential metal supply.

in California clearly shows that larger deposits tended to be discovered early in the exploration process (Chung, Singer, and Menzie, 1992). A direct consequence of this process is that the frequency distribution of sizes determined at an intermediate exploration stage overestimates the frequencies of large deposits and underestimates the frequencies of small undiscovered deposits. That is, the size distribution based on incomplete exploration of a region may be biased in favor of large undiscovered deposits. Tests of the discovery order versus sizes of other kinds of deposits are clearly warranted, but the analysis should be performed within the same exploration and geologic settings. For example, cases where exploration of the exposed bedrock is followed by exploration for deposits under cover should be considered independently because neither small nor large deposits in the subsurface would have been searched for in the initial stage of exploration. Care must also be exercised to account for extensions to reserves that may not be accounted for in the more recently discovered deposits.

From the preceding discussion, it is clear that most of the published grade and tonnage models include a significant proportion of noneconomic deposits and that, in most cases, low-grade or low-tonnage deposits (occurrences) not included in the models would have negligible effect on potential supply estimates. In the experience of most economic geologists, however, low-grade and particularly low-tonnage deposits (occurrences) are underrepresented in the models. The missing low-grade and small-tonnage deposits (occurrences) suggest that grade and tonnage models represent a biased sample of the large number of low-grade or small-tonnage occurrences and prospects found during exploration. This difference between the population represented by the grade and tonnage model and the population that may exist in the earth must be considered when the number of undiscovered deposits is estimated; this difference reflects the distinction between mineral deposits and mineral occurrences.

Some geologists have suggested that the grade and tonnage models should be extended to include not only deposits but occurrences. If the problem of possible biases due to incomplete exploration of these occurrences is neglected, then it is possible to construct such models; the tonnage model would, of course, have a much lower median. Because three-part assessments require that the estimated number of undiscovered deposits be consistent with the grade and tonnage model, the process of estimating the number of deposits might be more difficult because of the much larger number of "deposits" to be estimated. An economic analysis of the results of this assessment would show that the occurrences and probably some of the estimated undiscovered deposits would be uneconomic. Thus, including occurrences in the grade and tonnage model would require more work in the assessment while not affecting the final answer in any way.

When only one or two explored examples of a deposit type are known in a particular area, it is common to believe that they represent a special subtype or new type because they are almost never exactly the same as the "typical" deposit in every respect. Deposits will generally not have the median tonnage or grade of the type in question and may vary somewhat in mineralogy. To avoid the situation where every deposit is considered unique and therefore prediction is not possible, the well-explored deposits in an area should be tested to see if they are statistically different from the general model, as was done for two kuroko-type massive sulfide deposits in Oregon (Singer, in press a). If the well-explored (that is, completely drilled) deposits are significantly different in size or grade, then the local deposits should be examined to see if they belong to a geologically homogeneous subset of the original grade and tonnage model. Only if all of these conditions are met should a new submodel be constructed along with a consistent descriptive model. The revised model would then be used in conjunction with the estimates of the number of deposits.

Number of Undiscovered Deposits

The third part of an assessment is the estimate of some fixed, but unknown, number of deposits of each type that exists in the delineated tracts. Until the area being considered is thoroughly and extensively drilled, this fixed number of undiscovered deposits, which could be any number (including 0), will not be known with certainty.

Estimates of the number of deposits explicitly represent the probability (or degree of belief) that some fixed but unknown number of undiscovered deposits exists in the delineated tracts. As such, these estimates reflect both the uncertainty of what may exist and a measure of the favorability of the existence of the deposit type. Uncertainty is shown by the spread of the number of deposits estimates associated with the 90-percent quantiles to the 10- or 1-percent quantiles; a large difference in the numbers suggests great uncertainty. Favorability can be represented by the estimated number of deposits associated with a given probability level or by the expected number of deposits.

Estimates are by deposit type and must be consistent with the grade and tonnage model. Thus, the estimated number of deposits must match the percentile values of the grade and tonnage model. For example, at any level of probability, approximately half of the estimated undiscovered deposits should be larger than the median tonnage, and about 10 percent of the deposits should be as large as the upper 10 percent of the deposits in the tonnage model. If the grade and tonnage model is based on district data, like the Comstock epithermal vein gold-silver model (Mosier and others, 1986), then the number of undiscovered districts should be estimated. Some of the models were constructed with spatial distance rules such as the 500-m rule for combining mineralization in the kuroko massive sulfide model (Mosier and others, 1983). The same rule must be applied when the number of undiscovered deposits is estimated. Deposits in the study area that have published grades and tonnages are counted as discovered deposits; however, deposits without published estimates are counted as undiscovered to avoid double counting.

There are no fixed methods for making estimates of the number of undiscovered deposits. Several methods based on experience and logic, however, can be used directly or as guidelines to make these estimates. Each method represents some form of analogy. The most robust of these methods is a form of mineral deposit model wherein the number of deposits of each type per unit area from well-explored regions is counted, and the resulting frequency distribution is used either directly for an estimate or indirectly as a guideline in some other method. Although Allais (1957) employed this method of estimating the number of undiscovered deposits, many kinds of deposits were mixed together in his analysis. Only a few examples of this type of model that are consistent with descriptive and grade and tonnage models have been published (Bliss and others, 1987; Bliss, 1992b; Bliss and Menzie, in press); Cox (1993) describes an application of this model to estimate the number of gold veins in Venezuela. A variation of this method used linear regression to relate size of areas of ultramafic rock in California to the number of known podiform chromite deposits (Singer, in press b). This regression was applied to make probabilistic estimates of undiscovered podiform chromite deposits in southern Alaska (MacKevett and others, 1978) and in Costa Rica (Singer and others, 1987).

Local deposit extrapolations consist of projecting the deposit density (number of deposits per unit area) in a well-explored part of the study area to a less explored part of the study area. Typically, this method leads to a point estimate of the number of undiscovered deposits (Singer and MacKevett, 1977) unless assumptions can be made about an appropriate frequency distribution, such as described for mercury deposits in Alaska (Root and others, 1992). The base area need not be explored completely, but the number of deposits found and the proportion of the area explored must be estimated. A variation of this method takes advantage of information about the frequency distribution of tonnages for a deposit type and the extent to which the larger deposits in the study area have been discovered (Root and others, 1992).

Counting and assigning probabilities to anomalies and occurrences have long been practiced, but cases only recently have been documented (Reed and others, 1989; Cox, 1993). Either through experience or through statistical analysis of well-explored areas, the proportions of anomalies and occurrences that are actually mineral deposits can, in some cases, be estimated from the intensities and coincidence of different kinds of evidence.

A new method of providing guidelines that might be called "process constraints" has recently been proposed (Drew and Menzie, 1993). The basic premise is that the more likely the combination of geologic processes required for the formation of a deposit type, the more likely that the deposit type should occur. Thus, information about the processes that have occurred (or might have occurred) in a tract provides important information about the relative frequency of occurrence of deposit types.

Relative frequencies of related deposit types are valuable guides to estimating the number of undiscovered deposits (Drew and Menzie, 1993). For example, it is widely believed that where both polymetallic veins and porphyry copper deposits occur, the number of polymetallic veins is larger than the number of porphyry copper deposits. Thus, where both deposit types could occur in a permissive tract, we would expect the estimated number of polymetallic veins to be larger than the estimated number of porphyry copper deposits.

The sizes of alteration and mineralization zones around some deposit types are so large (Singer and Mosier, 1981a) that they can be used in some cases to set upper limits on how many deposits could exist in a delineated tract. These spatial limits have been informally used in some assessments.

The difference between the population of mineral deposits represented by the grade and tonnage model and the population of occurrences that may exist in the earth must be considered when the number of undiscovered deposits is estimated. The estimators must be certain that their estimates of the number of deposits are guided by a clear understanding of the corresponding grade and tonnage models. For the estimated number of deposits to be consistent with a grade and tonnage model, approximately half of the deposits estimated should have greater than the model's median tonnage or grade; in practice, grade is typically not of concern because even mineral occurrences often have grades similar to deposits. The requirement that half of the estimated deposits be larger than the median solves the most common estimation error, whereby the estimate incorrectly reflects the number of deposits that are larger than the lowest tonnage reported in the tonnage model. Estimates of the number of deposits must be consistent with the population of *mineral deposits* in the grade and tonnage model and not with the population of *mineral occurrences.*

In most three-part assessments, the final estimates have been made subjectively, often using one or more of the previously described methods as guidelines. A variety of different guidelines for estimates provides a useful cross-check of assumptions.

In practice, a small group of scientists who are knowledgeable about the deposit type (and advised by regional experts) typically makes consensus estimates. Two general strategies are used (Menzie and Singer, 1990): (1) individual occurrences, prospects, and indicators are assigned probabilities and the results combined; and (2) the estimator recalls from experience many other well-explored areas that are geologically similar to the area being assessed and uses the proportion of the areas having different numbers of deposits to make the estimates for the new area. In each case, the scientists must weigh all of the geoscience and exploration information. Until more estimation guidelines and density of deposits models are available, it seems prudent to rely on mineral deposit specialists to make subjective estimates because they can bring their experiences and observations to the process.

Subjective probabilities, such as those used here, have been variously called degrees of belief or propositional probabilities (Bacchus, 1990). The widest scientific application of this kind of estimation is in meteorology, where the reliability has been excellent: The relative frequency of occurrence of the event predicted is very close to the estimated probability (Murphy and Winkler, 1984). The oldest and probably most commonly practiced form of subjective estimation is gambling. For example, Stern (1991) showed that the distribution of actual margins of victory versus predicted point spreads in National Football League games has a mean of zero, indicating that nonscientists can also make unbiased subjective estimates. Geologists commonly make estimates that, although not explicitly quantitative, are subjective and have uncertainty, such as estimating locations of subsurface boundaries in geologic cross sections.

These examples from different fields demonstrate that, at least under some conditions, subjective estimates can be unbiased and reliable. The decades of experience of subjective and objective forecasting in meteorology provide insight into how the process of making subjective assessments in mineral resources might be improved. Murphy and Winkler (1984) found that consensus schemes performed better than almost all individual forecasters and that the best forecasts were made when objective forecasts were part of the information supplied to subjective forecasters. Among their recommendations were (1) more effective use of many information sources, (2) motivation to encourage forecasters to improve their performance, (3) formal procedures to assist forecasters in quantifying their uncertainty in terms of probability, and (4) quick and extensive feedback concerning performance. Quick and extensive feedback might be difficult to apply in mineral resource assessments, except possibly through training exercises.

The emphasis here on subjective estimation and the use of objective guidelines stems from the author's belief that objective quantitative methods have yet to be shown effective in estimating the number of undiscovered deposits. Three-part assessments are a form of a product, not a method, and therefore do not preclude the use of any method that is consistent with the other parts of the assessment.

Conclusions

The fundamental strength of the three-part quantitative assessment is its internal consistency. In three-part assessments, the estimates are internally consistent: Delineated tracts are consistent with descriptive models, grade and tonnage models are consistent with descriptive models, as well as with known deposits in the area, and estimates of the number of deposits are consistent with grade and tonnage models.

Consistency in the assessments is a direct consequence of the internal consistency required in the construction of the descriptive and the grade and tonnage deposit models. New models of the number of deposits per unit area and other quantitative extensions to the present models must also be consistent with the present models. That is, these new models must be constructed from deposits located in geologic settings that match the descriptive models and that are consistent with the appropriate grade and tonnage models. These new versions of deposit models, the quantification of models in general, and the development of guidelines or direct methods of estimation of the number of undiscovered deposits will all be successful to the extent that they are consistent with the other models used in assessments.

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